Beating the market: Can evolutionary-based portfolio optimisation outperform the Talmudic diversification strategy?

Safwan Mohd Nor

University of Malaysia Terengganu,
21030 Kuala Nerus,
Terengganu, Malaysia

and

Victoria Institute of Strategic Economic Studies,
Victoria University,
Vic 3000, Australia
Email: safwan@umt.edu.my

Sardar M.N. Islam*

Victoria Institute of Strategic Economic Studies,
Victoria University,
Vic 3000, Australia
Email: sardar.islam@vu.edu.au
*Corresponding author

Abstract: It is argued that with a small number of stocks (*N*) in a portfolio (which suits individuals rather than institutional investors), naive Talmudic diversification rule (1/*N*) offers a superior trading outcome against mathematically optimal portfolios due to its robustness against estimation error. As this puzzle has not been resolved, we explore it using an alternative portfolio choice problem that seeks to outperform the benchmark market index – FTSE Bursa Malaysia KLCI. This study makes a significant contribution by using an industry-common objective function and also incorporating floor/ceiling constraints and the effect of delisting. Using evolutionary algorithm, we construct optimal portfolios with varying *N*s in-sample for out-of-sample analysis. We find that 1/*N* is superior with smaller *N*s, although optimised portfolio dominates as *N* increases. However, with both diversification policies underperform the market and produce very low Sharpe ratios, their efficacies for practical applications are highly suspect.

Keywords: portfolio optimisation; Talmudic diversification policy (1/*N*); realistic trading constraints; individual; institutional investors.

Reference to this paper should be made as follows: Nor, S.M. and Islam, S.M.N. (2016) 'Beating the market: Can evolutionary-based portfolio optimisation outperform the Talmudic diversification strategy?', *Int. J. Monetary Economics and Finance*, Vol. 9, No. 1, pp.90–99.

Biographical notes: Safwan Mohd Nor is a Senior Lecturer at the University of Malaysia Terengganu, Malaysia and Research Associate at the Victoria Institute of Strategic Economic Studies, Melbourne, Australia. He holds a PhD in Finance from Victoria University, Australia and Master of Financial Analysis from La Trobe University, Australia. In addition, he has several years of industry experience as a stock market dealer. His main research interests lie in the areas of trading strategy, market efficiency, computational finance, financial modelling and optimisation.

Sardar M.N. Islam is a Professor at the Victoria Institute of Strategic Economic Studies, Victoria University, Australia. He has interest in different academic fields in Finance, Economics, Business, and Law where he has publications, expertise, or experience in teaching, supervision, and research. He has published extensively in the above areas. His qualifications include PhD and LLB. He has taught for more than 15 years at different universities in various countries. His research has gained international reputation.

This paper is a revised and expanded version of a paper entitled 'Beating the market: evolutionary-based portfolio optimization versus the Talmudic diversification strategy' presented at the *3rd International Conference on Management, Economics and Finance*, Kuala Terengganu, 27–28 October, 2014.

1 Introduction

Portfolio allocation, implying both efficient resource allocation and optimal investment planning, is a critical issue for both academic and investment practice. With trillions of dollars under management, mutual and pension funds are required to provide investors sufficient diversification in order to reduce overall portfolio risk. Ironically, however, large number of funds and investors follow the naive diversification rule – also known as the Talmudic or 1/*N* policy – instead of using the more sophisticated portfolio choice models (Benartzi and Thaler, 2001; Huberman and Jiang, 2006). Although optimised portfolio rules should, theoretically, outperform naive diversification, existing results remain inconclusive (Adame-García et al., 2015; Brown et al., 2013; Duchin and Levy, 2009; DeMiguel et al., 2009; Jacobs et al., 2014; Kritzman et al., 2010).

In this study, we contribute to this ongoing discussion by using an alternative portfolio optimisation model. Indeed, the focus of existing studies has primarily been on the classical (and extensions) of Markowitz mean variance model. Although active portfolio management attempts to outperform the market, such objective is not inherently built into the classical portfolio choice problem. Accordingly, we contribute by investigating a practical – but less researched – portfolio optimisation problem that seeks to outperform the benchmark market index. Using a sample of large Malaysian firms, we argue that for individual investors holding small number of stocks (*N*), naive portfolio dominates, while the benefit of optimised portfolio increases as *N* gets larger.

Existing research asserts that the superiority of either the mean variance optimisation model or 1/*N* strategy may depend on two factors:

- *N* in the portfolio
- stability of assets parameter over time.

The number of stocks in a portfolio is critical, as individual investors tend to hold small *N* as compared to those of institutional investors. In allocating investment capital, Duchin and Levy (2009) argue that a $1/N$ rule prevents incorrect portfolio mix caused by estimation errors of the parameters, but by definition it is suboptimal. However, the authors find that the $1/N$ loses its power of built-in hedge against estimation error as the number of stocks in the portfolio increases (see Levy and Duchin, 2010). In the existing literature, this issue of the determination of the superiority of these models has not been addressed adequately. In order to test for the two factors above, this study builds several portfolios with different *N*s and investigates the performance of the competing diversification strategies using out-of-sample data.

The contributions of this paper are as follows. First, to our knowledge, we are the first study to employ this unique objective function in comparing optimal with naive portfolios. Although studies have been recently undertaken to examine the relative performance between the optimised and naive portfolios, they tend to focus on the Markowitz model and/or its extensions. In practice, however, portfolio optimisation is not only restricted to Markowitz paradigm. In fact, large number of fund managers also attempts to outperform the market, and this objective is explored here. Second, we include both lower (floor) and upper (ceiling) bound constraints. Floor constraint avoids unnecessary costs with small holdings, while ceiling constraint limits excessive exposure to the portfolio. Third, we incorporate the effect of stock delisting. Unlike the asset holdings in Duchin and Levy (2009) and DeMiguel et al. (2009), individual stocks are exposed to the possibility of delisting. How post-optimisation delisting affects the remaining, surviving firms is critical and is addressed in this paper. Finally, we examine these models within an emerging market context. Prior studies in this area have only been explored in the developed markets. Emerging markets, however, have different characteristics which may result in different outcomes. For practitioners, these markets generally have low correlations with the developed ones (see Malkiel, 2007) and thus may be beneficial for investors seeking to diversify internationally. This justifies the application of the models and tests to the financial market in Malaysia, an emerging market.

Having outlined the relevant literature and how we make significant contributions, the remainder of this paper is structured as follows. Section 2 discusses the methodology of our study. Section 3 compares the performance between the evolutionary algorithm-based optimal and $1/N$ portfolios. Section 4 discusses the conclusion and implications.

2 Methodology

The data and models adopted in this study, motivated by the objectives of this research, are discussed in this section.

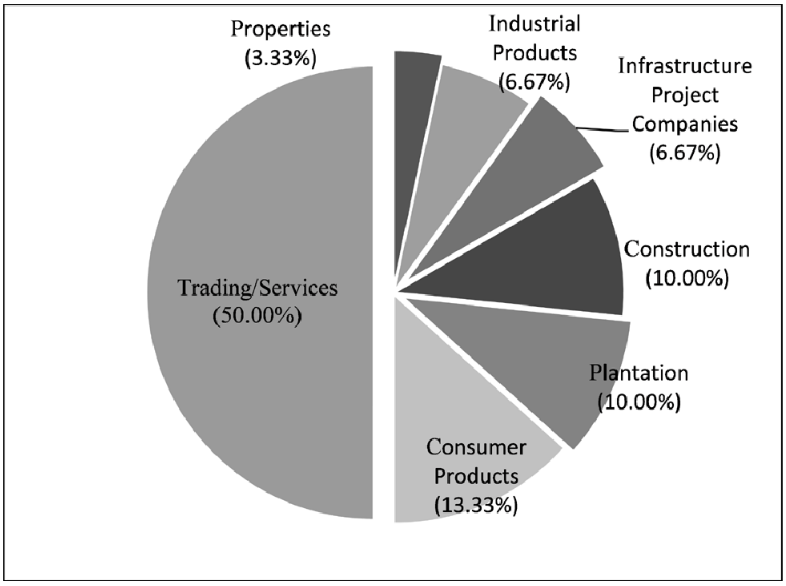
2.1 Portfolio data

We examine two portfolio models in the Malaysian stock market – the FTSE Bursa Malaysia. As argued by Loong (2006), portfolio optimisation is not well researched in Malaysia. Accordingly, the market provides an interesting platform to undertake this study as it mitigates the possibility of data mining bias. The analysis is performed on 30 nonfinancial firms selected from the FTSE Bursa Malaysia KLCI (FBM KLCI).[1] This sample comprises of the largest non-financial companies in the stock market by market capitalisation. The FBM KLCI is the main barometer for the Bursa Malaysia.

The use of these blue chips constituent firms indicates sufficient liquidity and tradability of stocks in the portfolios. In forming the dataset, we use the information available prior to the portfolio construction date. This mitigates any potential look ahead and survivorship biases. Historical data for the benchmark index and the firms are obtained from Thomson Reuters DataStream.

Following Duchin and Levy (2009), we construct four portfolios with different *N*s (15, 20, 25 and 30). Note that our sample size is limited to only 30 *N*s. It has been documented that any additional benefits from diversification are reduced when *N* becomes too large. This can be ascribed to many factors, such as the marginal reduction in variance as *N* increases and the additional costs incurred for monitoring and rebalancing the portfolios. Accordingly, several studies show that proper diversification can already be achieved when there are only about 15–30 stocks in the portfolio (for example, see Elton and Gruber, 1984; Reilly and Brown, 2003). Since proper diversification is not only concerned with allocation among stocks, but should also composed of stocks from different industries (Markowitz, 1952), our portfolio comprises of firms from diverse sectors. This is shown in Figure 1.

Figure 1 Industry composition of the portfolio



We divide the sample period into two non-overlapping sub periods. The first (in-sample) period ranges from 1st July, 2002 to 30th June, 2008 and is used for portfolio optimisation. The second, out-of-sample period ranges from 1st July, 2008 to 30th June, 2011 and is used for examining the performances of both optimal and naive rules.

2.2 Portfolio design

The two models investigated in this paper are as follows. For the first model, we construct an optimal portfolio using evolutionary algorithm (EA) with the goal of maximising the probability of beating the benchmark FBM KLCI. We call this model the 'evolutionary-based beating the market' portfolio model (EABM). We optimise EABM in-sample for out-of-sample forecasting and emphasise on the latter analysis, which is more relevant. This approach is consistent with Duchin and Levy (2009), DeMiguel et al. (2009), Levy and Duchin (2010). The second model is the naive 1/*N* portfolio. In this case, the investment funds are simply divided equally.

Consistent with real-life mutual funds in Malaysia (for example several funds offered by ING and OSK), the portfolios in this paper are rebalanced every quarter. We set the initial capital of RM100,000.00 in order to allow us measure the profitability of EABM and 1/*N* portfolios at the end of the period. While the objective of our study is to examine the probability of beating the market, it is interesting to see whether this will also lead to positive money (ringgit) returns. Indeed, as argued by Winston and Albright (2009), if the optimal portfolio often outperforms the market, good returns can be expected. In the spirit of examining portfolio performance, we also provide the annualised Sharpe ratios. Building upon Ragsdale (2008), Winston and Albright (2009), our portfolio selection problem can be mathematically presented as

$$\max\left(\frac{\sum_{t=1}^{T} x_t}{T}\right), \text{ subject to} \sum_{i=1}^{N} w_i = 1 \text{ and } l_i \le w_i \le u_i, \tag{1}$$

$$\text{where } x_t = \begin{cases} 1 & \text{if } \sum_{i=1}^{N} w_i r_{it} > B_t \\ 0 & \text{if } \sum_{i=1}^{N} w_i r_{it} \le B_t \end{cases} \tag{2}$$

in which T is the total (in-sample) period (quarters) ($t = 1, 2, 3, \ldots, T$), x_t is a binary variable which indicates whether the portfolio returns underperforms or outperforms the benchmark returns B_t at time t, w_i is the weight of stock i, r_{it} is the returns of stock i at time t, and l_i and u_i are the lower and upper bound weights of stock i respectively. We limit the floor constraint at 1%. For the ceiling constraint, we set it at 15% for $N < 20$ and 10% for $20 \le N \le 30$, consistent with Levy and Duchin (2010). At each quarter, the stocks in the portfolio are rebalanced to its optimal weights.

Since the portfolio is optimised only once using the in-sample data, any delisting of stocks in the holdout period requires adjusting the weights so that the initial, optimised weights corresponding between the remaining securities are preserved. To do this, we employ the method utilised in Aronsson et al. (2006), where the cash inflows from the delisted stocks will be reinvested to the remaining surviving stocks according to the optimised portfolio weights in the subsequent rebalancing period. In mathematical form, assuming that stock b is delisted, the new weight of stock a following rebalancing is given by:

$$w_{a'} = \frac{w_a}{\left(\sum_{i=1}^{N} w_i - w_b\right)}, \tag{3}$$

where $w_{a'}$ is the new weight of stock a after the delisting of b, w_a is the weight of stock a before the delisting of b, w_b is the weight of stock b before its delisting, w_i is the weight of stock i before the delisting of b, and N is the number of stocks in the portfolio before the delisting of b.

The optimisation problem in this paper involves computing the logical IF function. Accordingly, we use evolutionary algorithm to optimise the EABM portfolio that largely follows the methods discussed in Ragsdale (2008) and Winston and Albright (2009). First, we generate a population size of 200 that includes the initial set of chromosomes. Each chromosome represents the weight of stock i in the portfolio. Second, the EA creates a new generation. The chromosomes with larger fitness (objective) function possess greater probability of surviving into the next generation. This is done through two genetic operators, crossover and mutation. A pair of chromosomes generates offspring via crossover, in which the selected chromosomes are replaced by the offspring. The EA randomly modify members of the current population to produce new candidate solution. We set 0.5 as the probability of mutation operations. Mutations bring the optimisation to a completely different location in the feasible region and avoid the algorithms from becoming stuck. Finally, the stopping condition dictates how the process stops. At each generation, the best value of the fitness function is recorded and the algorithm repeats the second step. If there is no further improvement in the fitness function after several generations, the EA terminates.

3 Results and discussion

Table 1 reports the performance of both optimal (EABM) and naive ($1/N$) rules for the in-sample periods. As stated earlier, in-sample analysis is provided only for information purposes. By default, optimised portfolio will always be superior to the $1/N$ rule, or at least equal to it in cases where the Talmudic portfolio is already optimal.[2] This is evident by the number of times (quarters) of EABM portfolio outperforming $1/N$.

Table 1 In-sample analysis (1 July, 2002–30 June, 2008)

Number of stocks (N)	*Portfolio*	*Ringgit return* (RM)	*Annualised Sharpe ratio*	*Number of quarters beating the FBM KLCI*	*Number of quarters FBM KLCI superior*
15	EABM	174,954.89	1.22	18	6
	$1/N$	136,184.41	1.11	15	9
20	EABM	143,465.28	1.03	16	8
	$1/N$	115,616.04	0.93	14	10
25	EABM	95,660.20	0.76	14	10
	$1/N$	96,434.00	0.78	14	10
30	EABM	116,975.82	0.83	16	8
	$1/N$	123,931.98	0.85	15	9

Out of the six-year period ($T = 24$), the EABM is superior to $1/N$ when $N = 15$, 20 and 30, while at $N = 25$, both strategies beat the market 14 times. Throughout this period, both EABM and $1/N$ have greater number of quarters outperforming the FBM KLCI. EABM generates greater ringgit return and annualised Sharpe ratio compared to the $1/N$ portfolio at $N = 15$ and 20, while $1/N$ is more profitable and provide better risk-return tradeoffs when $N > 20$. It is worth noting that an investment of RM 100,000.00 at the beginning of the period in any of the Ns in EABM portfolios would result in substantial gain by 30th June, 2008, ranging from about 96% to 175%. Even by simply dividing funds equally would yield 96–135%. In addition to the overall Sharpe ratios which are within acceptable level, the in-sample results would probably entice investors to invest in the market (out-of-sample).

The more relevant, out-of-sample analysis, is provided in Table 2. During the holdout period, 1/*N* strategy dominates when there is small number of stocks ($N = 15$ and 20). As *N* increases to 25, both models have equal number of quarters outperforming the FBM KLCI (four out of eight). By the time $N = 30$, EABM dominates the 1/*N*. Consistent with Duchin and Levy (2009) and Levy and Duchin (2010), 1/*N* model is superior when *N* is small, but optimal portfolio dominates when *N* gets larger.

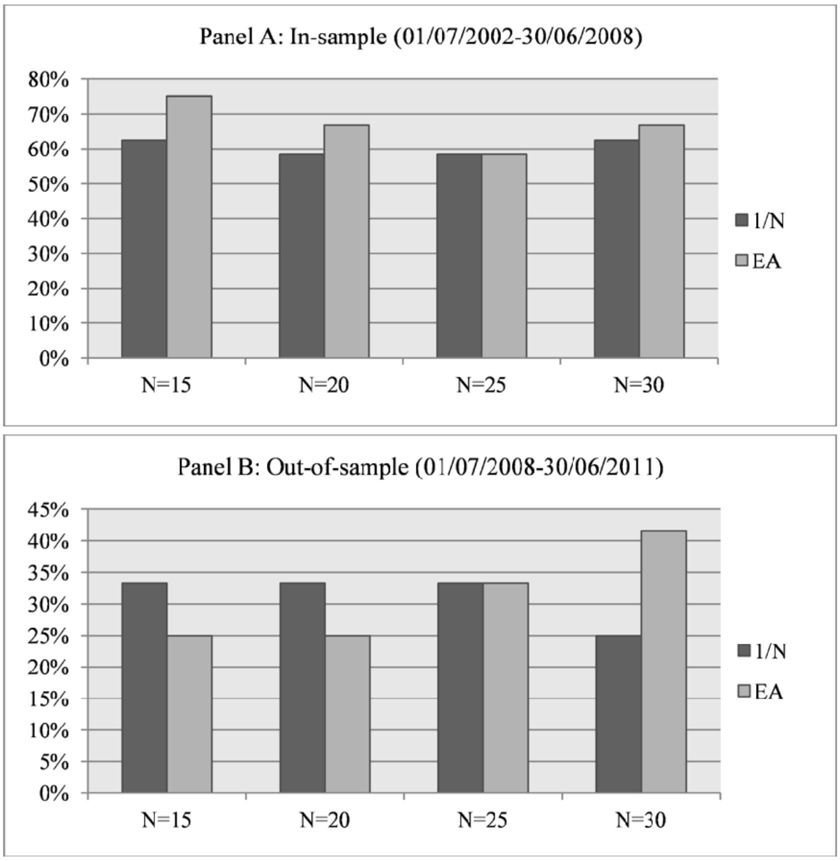
Table 2 Out-of-sample analysis (1 July, 2008–30 June, 2011)

Number of stocks (N)	*Portfolio*	*Ringgit return* (RM)	*Annualised Sharpe ratio*	*Number of quarters beating the FBM KLCI*	*Number of quarters FBM KLCI superior*
15	EABM	15,992.13	0.41	3	9
	1/*N*	12,906.30	0.34	4	8
20	EABM	18,714.27	0.44	3	9
	1/*N*	20,296.53	0.45	4	8
25	EABM	11,852.45	0.28	4	8
	1/*N*	15,520.22	0.35	4	8
30	EABM	19,037.91	0.37	5	7
	1/*N*	13,834.93	0.30	3	9

All of the portfolios examined generate returns ranging from about 12–20% by 30th June, 2011. However, annualised Sharpe ratios are well below the value of 1, with the highest of only 0.44 for EABM portfolio and 0.45 for Talmudic portfolio, both when $N = 20$. This signals very poor risk-return profile of both portfolios. However, as it is not the purpose of the portfolio selection model to maximise Sharpe criterion, main comparison should primarily be made on the basis of whether or not the EABM portfolio can indeed surpasses the market consistently, and also, how it compares with the Talmudic rule. Ironically, across all *N*s, none of the EABM (or even 1/*N*) strategy produce greater number of times beating the FBM KLCI. In hindsight, one would be better to invest in the index (or index fund) rather than following the optimal or naive rule.

The relative performance between the optimised and Talmudic portfolios can be seen in Figure 2. The charts show the probability (in percentage) of beating the market benchmark for both periods across different number of stocks. As described, while both strategies are superior to the benchmark returns most of the quarters in-sample, none of the models is superior to the market during the out-of-sample period. Nonetheless, it is evident that in comparing 1/*N* with EA-based portfolios, the former has greater probability of beating the market when *N* is small, but the latter is superior with larger number of stocks in the portfolio, supporting previous studies.

Figure 2 The probability of beating the benchmark FTSE Bursa Malaysia KLCI



4 Conclusion and implications

It has been argued that the relative performance of optimal and naive diversification policies can be attributed to the number of securities available in the portfolio and also the possibilities of estimation error. We contribute to the current, hotly contested topic of optimal vs. Talmudic allocation strategy by investigating the optimisation of an alternative and simple objective function – maximising the probability of beating the market. We find that, for individual investors holding small number of securities ($N < 25$), naive diversification dominates. Alternatively, with large number of securities ($N > 25$), portfolio optimisation dominates. Our results imply that $1/N$ is robust against estimation error at small N. This $1/N$ dominance at small Ns provides support to the naive strategy's built-in hedge against estimation error when the number of stocks in the portfolio is small, corroborating Levy and Duchin (2010). As N increases, its ability diminishes.

In terms of practical implications, both strategies have positive ringgit returns. Nevertheless, the fact that EABM and $1/N$ portfolios yield very low Sharpe ratios and consistently being outperformed by the FBM KLCI indicate that these strategies are not viable and also risky. As such, the results suggest that both individual and institutional investors should steer clear of these portfolio allocation policies. Briefly stated, our findings are consistent with Winston and Albright's (2009) argument that a portfolio optimised on the goal of outperforming the market does not necessarily offer an acceptable risk level. Future research can investigate the performance of naive vs. optimal portfolios by incorporating other constraints, such as complex round-lot and trading costs. Different portfolio selection models and performance measures (Nguyen, 2014), as well as exploring Shariah-compliant funds (Kok et al., 2009) for the construction of optimal and 1/*N* portfolios, also provide interesting scope for further studies.

Acknowledgement

We are grateful for the comments and suggestions made by Professor Imad Moosa (RMIT University), Professor Xiaoxia Huang (University of Science and Technology Beijing) and Professor Wei Li (Hong Kong Polytechnic University). All remaining errors are our own.

References

Adame-García, V.M., Fernández-Rodríguez, F. and Sosvilla-Rivero, S. (2015) *Portfolios in the Ibex 35 Index: Alternative Methods to the Traditional Framework, A Comparative with the Naive Diversification in a Pre- and Post-Crisis Context*, The Complutense University of Madrid, Working Paper No. 1507, June, pp.1–32.

Aronsson, A.O., Kirova, K. and Yang, C. (2006) *Asset Management of the Foundations of the City of Göteborg*, Master Thesis, Göteborg University.

Benartzi, S. and Thaler, R.H. (2001) 'Naive diversification strategies in defined contribution saving plans', *American Economic Review*, Vol. 91, pp.79–98.

Brown, S.J., Hwang, I. and In, F. (2013) 'Why optimal diversification cannot outperform naive diversification: evidence from tail risk exposure', *European Financial Management Association (EFMA) 2013 Annual Meetings*, 26–29 June, Reading, UK, pp.1–55.

Demiguel, V., Garlappi, L. and Uppal, R. (2009) 'Optimal versus naive diversification: how inefficient is the 1/N portfolio strategy?', *Review of Financial Studies*, Vol. 22, pp.1915–1953.

Duchin, R. and Levy, H. (2009) 'Markowitz versus the Talmudic portfolio diversification strategies', *Journal of Portfolio Management*, Vol. 35, pp.71–74.

Elton, E.J. and Gruber, M.J. (1984) *Modern Portfolio Theory and Investment Analysis*, John Wiley & Sons, New York.

Huberman, G. and Jiang, W. (2006) 'Offering versus choice in 401(k) plans: equity exposure and number of funds', *Journal of Finance*, Vol. 61, pp.763–801.

Jacobs, H., Müller, S. and Weber, M. (2014) 'How should individual investors diversify? An empirical evaluation of alternative asset allocation policies', *Journal of Financial Markets*, Vol. 19, pp.62–85.

Kok, S., Giorgioni, G. and Laws, J. (2009) 'Performance of Shariah-compliant indices in London and NY stock markets and their potential for diversification', *International Journal of Monetary Economics and Finance*, Vol. 2, pp.398–408.

Kritzman, M., Page, S. and Turkington, D. (2010) 'In defense of optimization: the fallacy of 1/*N*', *Financial Analysts Journal*, Vol. 66, pp.1–9.

Levy, H. and Duchin, R. (2010) 'Markowitz's mean–variance rule and the talmudic diversification recommendation', in John, B. and Guerard, J. (Eds.): *Handbook of Portfolio Construction: Contemporary Applications of Markowitz Techniques*, Springer, Anchorage.

Loong, C.S. (2006) *Portfolio Optimization and Value-Weighting – The Malaysian Context*, Doctor of Business Administration Thesis, University of South Australia.

Malkiel, B. (2007) *A Random Walk Down Wall Street: The Time-Tested Strategy for Successful Investing*, W.W. Norton & Company, New York.

Markowitz, H. (1952) 'Portfolio selection', *Journal of Finance*, Vol. 7, pp.77–91.

Nguyen, T. (2014) 'Selection of the right risk measures for portfolio allocation', *International Journal of Monetary Economics and Finance*, Vol. 7, pp.135–156.

Ragsdale, C.T. (2008) *Spreadsheet Modeling & Decision Analysis*, Thomson South-Western, Mason.

Reilly, F. and Brown, K. (2003) *Investment Analysis and Portfolio Management*, 7th ed., Thomson South-Western, Cincinnati.

WFE (2009) *Annual Report and Statistics 2009*, World Federation of Exchanges, Paris.

Winston, W.L. and Albright, S.C. (2009) *Practical Management Science*, South-Western Cengage Learning, Mason.

Notes

[1]The list of companies is obtained from the Bursa Malaysia website (http://www.bursamalaysia.com). Restructuring in the market on 6 July 2009 saw the FTSE Bursa Malaysia KLCI constituents reduced to only 30 firms, while the largest 100 companies in the market is now known as the FTSE Bursa Malaysia 100. During that year, the total number of listed companies was 959, as reported by the World Federation of Exchanges (WFE, 2009). Banking and financial services firms are ignored as these companies are highly regulated and have different financial reporting requirements.

[2]Since the portfolio is optimised in-sample, detail comparison against the naive portfolio using that same dataset is irrelevant. In any study on optimisation, the true performance of the portfolios must be measured using a separate, previously unseen dataset (i.e., out-of-sample data).