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## Original Research Paper

# Outflow of traffic from the national capital Kuala Lumpur to the north, south and east coast highways using flow, speed and density relationships

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## ABSTRACT

The functional relationships between flow (veh/km), density (veh/h) and speed (km/h) in traffic congestion have a long history of research. However, their findings and techniques persist to be relevant to this day. The analysis is pertinent, particularly in finding the best fit for the three major highways in Malaysia, namely the KL-Karak Highway, KL-Seremban Highway and KL-Ipoh Highway. The trans-logarithm function of density–speed model was compared to the classical models of Greenshields, Greenberg, Underwood and Drake et al. using data provided by the Transport Statistics Malaysia 2014. The results of regression analysis revealed that the Greenshields and Greenberg models were statistically significant. The trans-logarithm function was also tested and the results were nonetheless without exception. Its usefulness in addition to statistical significance related to the derived economic concepts of maximum speed and the related number of vehicles, flow and density and the limits of free speed were relevant in comparing the individual levels of traffic congestion between highways. For instance, KL-Karak Highway was least congested compared to KL-Seremban Highway and KL-Ipoh Highway. Their maximum speeds, based on three lanes carriage capacity of one direction, were 33.4 km/h for KL-Karak, 15.9 km/h for KL-Seremban, and 21.1 km/h for KL-Ipoh. Their corresponding flows were approximated at 1080.9 veh/h, 1555.4 veh/h, and 1436.6 veh/h.

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## 1. Introduction

Commuting traffics in the Federal Territory of Kuala Lumpur and the Klang Valley in general, are worsening every day despite efforts on speed-up fast lanes at the highway tolls. The Smart Tag and Touch-n-Go, in addition to several ordinary lines have been replicated but congestion continues to persist. Alternative models of transportation including light rail-transit, commuter and fast track lanes to help self-driven automobiles from congestion had already been implemented. On the supply side, these are the priority agendas of transport policy to ease traffic congestion. The problem of traffic congestion around the Klang Valley is escalating its impact to the other major cities in Malaysia. On the demand side, at least two reasons are relevant for this problem. First, the automobile industry is soaring every year with new and attractive models. The industry and its technology are past developing due to competition on the global business with the major producers in 2014 in China (23,722.8 thousands units), the United States of America (11, 660.7 thousands units), Japan (9774.6 thousands units), Germany (5907.5 thousands units) and South Korea (4524.9 thousands units) (OICA, 2015). Malaysia's total production was 596.6 thousands units. It was not in the list of big players for it is a small country. Increased urban population, higher per capita income and availability of credit for automobile purchases enable people to own cars (Almselati et al., 2011). Second, trade and economic activities in the country are booming and land transports like trailers, trucks carrying heavy shipments of timbers, petroleum and merchandised goods, and commercial vehicles are common scenarios on the highways.

This study will focuses on the traffic flow on the three major highways starting from Kuala Lumpur (KL) to the east coast, southern and northern states of the country, namely the KL-Karak Highway, KL-Seremban Highway and KL-Ipoh Highway known as part of North–South Expressway. The lengths of these specific surveyed highways were 19, 8.1, and 12.1 km, respectively (Ministry of Transport, Malaysia). With reference to statistics of one direction traffic flow for 2013, the average daily traffic for KL-Karak were 147.4 thousands, 206.1 thousands for KL-Seremban, and 253.2 thousands for KL-Ipoh. The growth rates for the five-year period of 2008–2013 were estimated at 1.3 percent, 2.2 percent and 9.5 percent, respectively. Evidently the KL-Ipoh Highways had illustrated a dramatic upsurge in the total number of traffics commuting to the northern regions over the last five years while traffics traveling to the east coast was the lowest. The east coast regions are less developed compared to the west coast areas by comparing the volume of traffics on the North–South Expressway.

The salient issue associated with traffic congestion is the problem of environmental degradation and the health deterioration of the society at large due to increased carbon dioxide emission. Carbon dioxide emission caused by traffic congestion is an external cost to the society which should be accounted by the automobile industry. In addition to the private cost, the social cost that the society bears in terms of the polluted air, environmental degradation, the worsening of health and productivity, will subsequently increase industrial

cost. A higher cost normally leads to a reduction in the industry automobile that could ease traffic congestion.

The other pertinent issue associated with traffic congestion and directly felt by the commuters is the loss of time causing delay to reach the destination. Delay due to traffic congestion is unpredictable and the worse congestions occur following fatal accidents. The sharp increase in number of vehicles contributes to congestion problem and the probability of the automobile accidents. Several studies would be interested to know how much commuters are willing to pay for avoiding traffic congestion by internalizing the cost of congestion to the responsible commuters. The demand for a reduction in traffic congestion reflects the consumer's awareness for the improved condition of highway service.

The study is a part of the internalization of congestion cost survey using the contingent valuation methods (CVM) conducted in November–December 2014. The current investigation is deemed complementary to the results of the above survey and can be utilized inter alia with its findings. Specifically, the objective is to establish the functional relationship between traffic flow measured in vehicles per hour and the mean-speed measured in kilometers per hour to estimate the maximum speed ( $v_M$ ) and maximum flow ( $q_M$ ) for comparative analysis about their degree of congestion. An estimate of density measured in vehicles per km versus speed (km/h) using various functional forms that fit Malaysia traffic characteristics in relation to the theoretical traffic congestion models estimated earlier by pioneer researchers in this field. The classical traffic congestion models were applied to the Malaysian case hoping that useful parameters and specific concepts of interest derived from the analysis can be inferred. The trans-logarithm (interchangeably used as trans-log) models will be compared with these classical models in terms of their statistical robustness in explaining the current analysis of traffic data relating to the traffic congestion of the selected highways.

## 2. Literature review

Several investigations have been conducted in Malaysia pertaining to problem of traffic congestion primarily related to the issues that are pertinent to the ongoing problems encountered by the highway commuters. One of the congestion hypotheses is that commuters prefer private automobiles to the public transport, specifically buses. Thus any study on public transport service and consumer satisfaction may shed light on the issue (Hamzah and Azli Ayub, 2015; Ismail et al., 2012). Hamzah and Azli Ayub revealed that the value invested by the service providers instead of their quality and reputation was the major influence on consumer satisfaction. The public transport service can never be claimed efficient in terms of value of time punctuality due to traffic congestion. Mahirah et al. (2015) investigated highways users' willingness to pay to reduce traffic congestion. The value of willingness to pay was estimated in terms of the highway's toll ticket. Assessment of traffic noise had also been investigated as a result of traffic congestion (Aziz et al., 2012). The noise level was higher during the peak hours particularly towards the afternoon and the sound pressure levels (SPLs) had shown a significant correlation with the distance from the highway areas. With

reference to the cited studies, the present investigation on speed, flow and density on highway traffic congestion is obviously an important research.

Literature reviews on the theoretical development of road traffic congestion can be classified in two categories which are static and dynamic models. Static models are time-independent and although have obvious limitations, which are still important for the analysis of the bird's-eye view perspective of traffic congestions before further understanding of a more complicated and realistic dynamic models.

Numerous references on the application of the mathematical models related to the fundamental relations of flow, speed and density were found in the literature. Greenshields et al. (1935) derived a linear speed–density relation with the assumption of uninterrupted flow, leading him to propose an approximate parabolic function of speed–flow and flow–density. Another speed–density relation was proposed by Greenberg (1959) using a logarithmic function and by Underwood (1961) using the exponential form. Afterwards, several researches on single models of speed–density expanded with revision and improvement as proposed by Pipes (1967), Newell (1961), Del Castillo and Benitez (1995), Jayakrishnan et al. (1995), Kerner and Konhäuser (1994), Van Aerde (1995), MacNicholas (2008) and Wang et al. (2011). Edie (1961) was the first one to perform a two-regime model which used two different curves to model free-flow and congested-flow. He identified a discontinuity in speed–density when he combined Greenberg's free-flow model and Underwood's congested model. The multi-regime model was then further explored by May (1990) and Sun and Zhou (2005).

Multiple investigations on the application of traffic free-flow and congested models were then performed with real field data in order to obtain a measure of the model capability to represent information from different sites. Hall et al. (1986) had analyzed the flow–density relationship of Ontario Highway in Canada, and Olszewski et al. (1995) had collected data to analyze the specific roads in Singapore using travel time and the volume of traffics. Nielsen and Jorgensen (2008) investigated the speed–flow and flow–density associations for the highway network of Copenhagen in Denmark. Lu and Meng (2011) had applied various steady-state mathematical models developed for analyzing traffic congestion patterns for the Beijing Third Road and the Jing-Jin-Tang Highway in China. Erlingsson et al. (2006) reported Van Aerde as the best fit in modeling the two urban highways, an arterial and a rural highway in Iceland. A comparison of the methodologies and results of the investigation were made with respect to their appropriateness in predicting variables useful for traffic policy specifications.

The work describing here attempts to improve upon prior models of flow–speed and speed–density relations in ways that are more applicable in real conditions and fundamentally consistent with actual data.

### 3. Background of data

Results of the regression analyses presented in this study were obtained from secondary data of annual average daily traffic (AADT) for 63 locations in Peninsular, Malaysia,

provided by the Ministry of Transport, Malaysia. Analyses were performed on three selected major highways in North–South Expressway namely KL-Seremban Highway (E37, KM 8.1), KL-Ipoh Highway (KM 8.1), and the KL-Karak Highway (E8, KM 19). The annual traffic flow was converted to the daily average traffic by dividing the total annual traffic with 365 d. The authors have assumed that the collection of traffic count was recorded for every one minute for the given length of highway. The formula used in estimating the traffic flow ( $q$ ) in terms of vehicles per hour was as  $q = (3600N/60)/24$ , where  $N$  is the number of vehicles (Fundamentals of Transportation, 2009). The density measured in vehicles per kilometer, was an approximation of the daily traffic over the length of highway surveyed. Flow should be equal to density multiplies by speed (Sharma et al., 2012).

The locations of the three highways are shown in Fig. 1. An average number of vehicles per day that passes through these highways were 147,447 for KL-Karak, 206,140 for KL-Seremban and 216,424 for KL-Ipoh. KL-Seremban and KL-Ipoh run through Kuala Lumpur which is the central business district, thus occupying a higher number of average vehicles per day. The heavy traffic period of KL-Seremban and KL-Ipoh are during weekdays, morning and evening as people use them for commuting back and forth from home to work. The main vehicles are cars, trucks, busses and motorcycles. All three highways are tolled.

Table 1 shows the estimates of one-way flow, density and speed for the KL-Karak Highway for the selected years. ADT is average daily traffic of three lanes in one flow direction from Kuala Lumpur to Karak. By definition  $q = vk$ , speed refers to speed adjusted to the annual growth rate of total vehicles that affect congestion. Speed refers to the adjusted mean speed which is the actual speed obtained by dividing the flow by the density plus or minus the annual rate of growth of the total traffic for that particular year. The current analysis of traffic congestion would yield useful insight with the availability of actual traffic data and the limitation on number of observations could have been improved for better results. The objective of this investigation is to fill-in the gap on theoretical analysis of traffic congestion from the economic perspective.

## 4. Methodology for traffic congestion

The flow–speed function and density–speed function were estimated for the three highways in order to find out which forms were most appropriate from statistical analyses. The flow–speed function utilizes the trans-log equation while the density-function uses the classical as well as the trans-log equations. Their statistical measurements and parameters will be presented to aid decision in evaluation of the estimated models.

### 4.1. Trans-logarithm function

Flow is chosen as the dependent variable and speed as the independent variable because a variation in vehicle's speed is responsible for the variations in flow. The piling of vehicles always happens at the traffic light, bottleneck junction, an

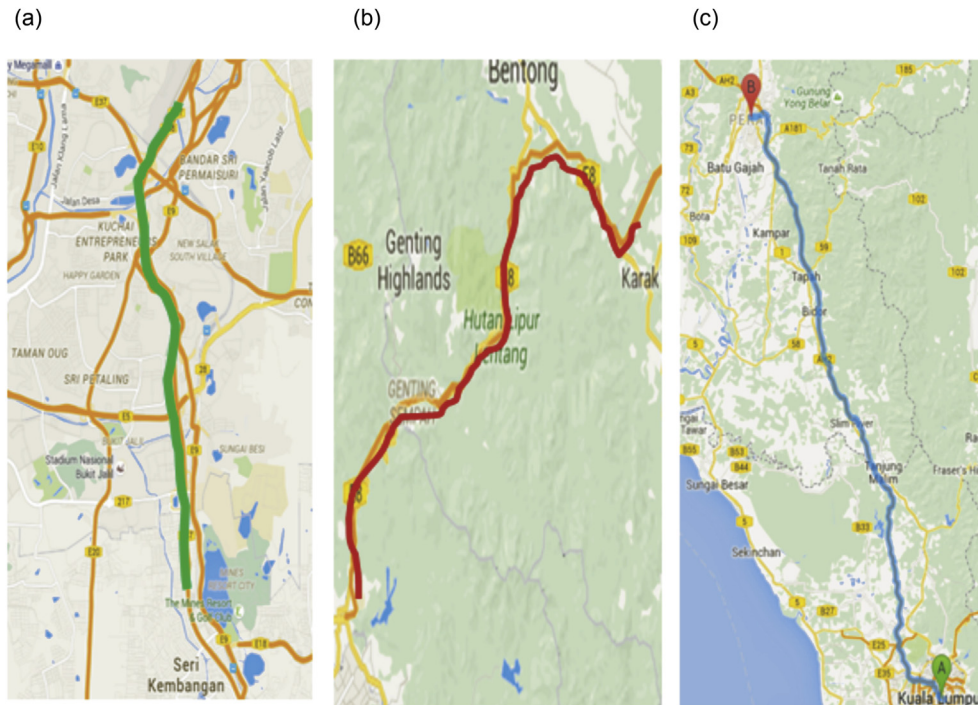


Fig. 1 – Road map for selected highways. (a) KL-Seremban (b) KL-Karak (c) KL-Ipoh.

entrance of traffic flow into the existing highway and any scene of accidents. In the initial stage as vehicle's speed slow down owing to traffic blockage the flow gets accumulated. Traffic congestion begins to reach the point of a maximum speed ( $v_M$ ). After reaching the maximum speed,  $v_M$ , traffic starts to ease out, speed begins to pick-up again and flow decline accordingly. This form of flow–speed function can be mathematically depicted by the quadratic function or the polynomial of power two or probably by the trans-logarithm function.

The trans-log is a form of quadratic equation using logarithm which is written mathematically for a single input–output relationship without a constant as follow

$$\ln(q) = \beta \ln(v) + \gamma [\ln(v)]^2 \quad \gamma < 0 \tag{1}$$

where  $q$  is the flow (measured in vehicles per hour),  $v$  is speed (measured in kilometer per hour).

The maximum speed,  $v_M$ , is obtained after setting  $\frac{\partial \ln(q)}{\partial \ln(v)} = 0$  and  $\ln(v_M) = \frac{\beta}{2\gamma}$  alternatively,  $v_M = e^{\beta/2\gamma}$  and the natural logarithm of maximum flow,  $\ln(q_M) = \frac{\beta}{2\gamma} \left( \beta - \gamma \frac{\beta}{2\gamma} \right)$  alternatively,

$q_M = e^{\beta(\beta - \gamma\beta/2\gamma)/2\gamma}$ . In addition to these useful traffic flow and speed concepts we could also approximate the maximum free speed at which  $\ln(q) = 0$ , which is  $\ln(v) = \frac{\gamma}{\beta}$  subsequently,  $v = e^{\gamma/\beta}$ . Eq. (1) can be transformed to  $\ln(q) = \beta \ln(v) \left[ 1 - \frac{\gamma}{\beta} \ln(v) \right]$ .

#### 4.2. Classical mathematical functions

The density–speed function was also estimated using a single input–output trans-log function. The models chosen were those used in Lu and Meng (2011) in their studies of the Beijing Third Ring Road and Jing-Jin-Tang Highway. These classical models of traffic congestion are Greenshields et al. (1935), Greenberg (1959), Underwood (1961) and Drake et al. (1967). The authors have derived and presented the necessary characteristics of each classical model to assist discussions. This is shown in Eq. (2) for the Greenshields model and the rest are shown in Eqs. (3)–(5).

$$k = k_j \left( 1 - \frac{v}{v_f} \right) \tag{2}$$

Greenshields denotes that  $k_j$  is the density at jam/congested condition. This means when speed is reduced to zero,  $v = 0$ , then  $v/v_f = 0$  and hence density  $k = k_j$ . Free velocity ( $v_f$ ) is the maximum speed cannot be greater than  $v$ . The proportion of  $v/v_f$  reflects the level of traffic congestion, the higher the value of  $v_f$  relative to  $v$ , the lower is the condition of congestion. After the level of  $v_f$  congestion tends to decline. The variables needed for regression estimation are  $k$  and  $v$  and it can be estimated using a linear regression equation.

The Greenberg density function has an exponential relationship with the speed. This is shown in Eq. (3).

**Table 1 – Estimate of three-line flow, density and speed for KL-Karak Highway, Malaysia, for specific years within 2001–2013.**

Year	AADT (veh)	ADT (veh/d)	Flow ( $q$ ) (veh/h)	Density ( $k$ ) (veh/km)	Speed ( $v$ ) (km/h)
2001	159,309	436.5	1091.16	22.97	47.50
2005	146,831	402.3	1005.69	21.17	44.90
2009	130,209	356.7	891.84	18.78	50.29
2013	147,449	404.0	1009.92	21.26	48.80

$$k = k_j \exp\left(-\frac{v}{v_0}\right) \tag{3}$$

For ease of explanation Eq. (3) can be transformed to  $\ln(k) = \ln(k_j) - \frac{v}{v_0}$ . When speed falls to zero, then  $\ln(k) = \ln(k_j)$  or  $k = k_j/e^0$  for  $e^0 = 1$ , which means density reaches a blockage condition. Again the intensity of  $v/v_0$  determines the level of traffic congestion, in that, a ratio of 0.5 implies  $k = k_j e^{-0.5} = k_j/e^{0.5}$  for  $e = 2.718282$ . A higher ratio of  $v/v_0$  increases the congestion level and vice-versa.

The Underwood density–speed equation for traffic congestion is presented in natural logarithm which is as follow

$$k = k_0 \ln\left(\frac{v_f}{v}\right) \tag{4}$$

This function illustrates the case when  $v_f/v = 1$ ,  $\ln(1) = 0$ , then  $k = 0$ , which means the density (vehicles per hour) is idle from the commuting vehicles during the period, that is, the condition of the highway is totally uncongested since there is no driver passes the highway. However, as the ratio increases to  $v_f/v = 2.718282$ ,  $\ln(2.718282) = 1$ , then  $k = k_0$ , the density of vehicles per hour increases to the maximum density. The last mathematical model developed by researchers to explain the density–speed relationship is the extension of Drake et al. model as shown below.

$$k = k_0 \left[2 \ln\left(\frac{v_f}{v}\right)\right]^{1/2} \tag{5}$$

This traffic congestion model is somewhat identical to the above Eq. (4) although the degree of variation of speed on density may differ but the pattern is quite similar except for the square-root and the doubling of logarithm for the  $v_f/v$  ratio.

## 5. Results and discussion

The results of fixing flow against speed in regression analysis will be presented in Section 5.1 and the density–speed relationship will be discussed in the following section. Section 5.2 focuses on density–speed analysis that is fundamental to all classical models. A comparison of these classical with trans-log model will be made to assess the strengths and weaknesses of their basic statistical attributes.

### 5.1. Analysis of flow–speed modeling using trans-logarithm function

Table 2 shows results of the regression analysis of traffic flow against speed for the three highways; KL-Karak, KL-

Seremban and KL-Ipoh. Evidently, all the trans-logarithmic models are statistically acceptable in explaining the relationship of the flow–speed relationships in traffic congestion. The  $R^2$  coefficient of KL-Seremban Highway is apparently lower showing that it is not the best fit. All estimates of  $R^2$  coefficient for KL-Seremban irrespective of their mathematical forms are relatively lower which means that additional observations of data are needed. The current study has no access to this information as such is constrained by additional data points. In general, the overall variations in speed have sufficiently explained the variations in traffic flow. 93.2 percent (KL-Karak), 83.0 percent (KL-Ipoh) and only 56.7 percent (KL-Seremban) variations in traffic flow were explained by the traffic speed, respectively. However, all the individual t-values are statistically and highly significant at 0.01 probability level. Hence, the mathematical models chosen are statistically appropriate and acceptable. Except for KL-Karak Highway, the value of Durbin–Watson apparently exhibits certain degree of autocorrelations between the error terms and the dependent variables.

In Table 2, the characteristics of relationship between traffic flow and speed in logarithm scale begins at the origin. A negative sign for the second term  $[\ln(v)]^2$  shows that there exists a maximum point for  $v$  and  $q$  (in logarithm scale). This corresponds to a congested traffic condition whereby as speed starts to increase congestion accumulates to a maximum traffic flow. Beyond this maximum level jam begins to ease off gradually to a free speed and as traffic congestion starts to fall, flow begins to decline swiftly. This table also shows the speed level in logarithm scale and the actual speeds in large brackets when traffic flows were set at zero. These figures apparently witnessed the maximum length of free speed when the flows are absent from congestion with KL-Karak, which have the highest length in kilometer followed by KL-Ipoh and KL-Seremban.

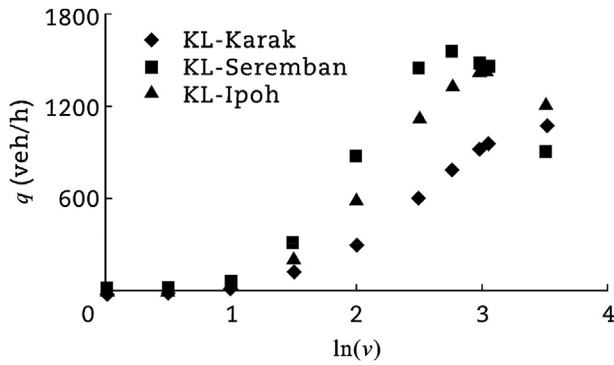
Fig. 2 illustrates the flow–speed curves for the three highways. At the initial part of the plot, traffic congestion accumulates at its highest level as demonstrated by the minimum speed and flow of the traffic. As speed rapidly increases, flow also begins to steadily increase with a lower rate than speed until it reaches an inflation point where the rate of flow increases and exceeds the speed. Speed of the highways begins to accumulate as speed accelerated to a maximum level. After reaching the maximum speed, the maximum flow starts to decrease gradually following an increase in speed level.

Table 3 shows the actual level of the maximum speed and flow of the three highways. Traffic congestion begins at the early stage of the flow–speed relationship. The KL-Seremban Highway shows the lowest maximum speed of

**Table 2 – Results of regression analyses of traffic flow (q) and speed (v).**

Highway		$\ln(v)$	$[\ln(v)]^2$	$\ln(v)$ & $[v \text{ (km/h)}]$ when $\ln(q) = 0$	$R^2$
KL-Karak	$\ln(q)$	3.9828 (85.97)***	-0.56770 (-49.130)***	7.016 [1114.0]	0.932
KL-Seremban	$\ln(q)$	5.3156 (25.09)***	-0.96114 (-13.790)***	5.531 [252.3]	0.567
KL-Ipoh	$\ln(q)$	4.7654 (19.470)***	-0.78091 (-12.330)***	6.102 [446.9]	0.830

Note: “\*\*\*” is significant at 0.01 for t-value in parentheses (small brackets).



**Fig. 2 – Flow and speed relationship in logarithm scale for Malaysia highways.**

15.883 km/h because of the congestion with the highest flow estimated around 1555.415 veh/h (Table 3). The speed for KL-Karak Highway is the highest among the three highways, 33.376 km/h, and hence it has the lowest flow of 1080.852 veh/h. For KL-Ipoh, the maximum speed is 21.140 km/h, and its flow is close to that of the KL-Seremban 1436.624 veh/h. From the traffic congestion perspective, the KL-Karak Highway is the least congested route. The KL-Seremban Highway is the most congested route while KL-Ipoh Highway has become more congested in recent years.

**5.2. Regression models of speed–density using classical methods**

The best fitted steady-state mathematical models of traffic congestion estimated from the density–speed regression analyses of the current data were obtained from the Greenberg (1959) and Greenshields et al. (1935) methods which were presented in Tables 4 and 5. Evidently, the estimated density for the Greenberg model were 32.095 veh/h for the KL-Karak Highway, 117.942 veh/h for KL-Seremban Highway and 72.017 veh/h for KL-Ipoh Highway.

From Table 4, density could be expressed as follow

$$k = \begin{cases} 32.095 \exp\left(-\frac{v}{105.437}\right) & \text{KL-Karak} \\ 117.942 \exp\left(-\frac{v}{41.551}\right) & \text{KL-Seremban} \\ 72.017 \exp\left(-\frac{v}{58.401}\right) & \text{KL-Ipoh} \end{cases}$$

From Table 5, density could be written as follow

$$k = \begin{cases} 27.318 \left(1 - \frac{1}{195.982}v\right) & \text{KL-Karak} \\ 111.36 \left(1 - \frac{1}{58.298}v\right) & \text{KL-Seremban} \\ 67.67 \left(1 - \frac{1}{88.227}v\right) & \text{KL-Ipoh} \end{cases}$$

The estimated density figures obtained from the Greenshields model were 27.318, 111.360 and 67.670 veh/h, respectively. The constant terms for all the highways were significantly different from zero at 0.01 probability level and the coefficients of the speed variable in both cases were significant at 0.01 (KL-Karak) and 0.05 (KL-Seremban and KL-

**Table 3 – Maximum speed and flow for selected highways.**

Highway	$\ln(v_M)$	$\ln(q_M)$	$v_M$ (km/h)	$q_M$ (veh/h)
KL-Karak	3.508	6.986	33.376	1080.852
KL-Seremban	2.765	7.350	15.883	1555.415
KL-Ipoh	3.051	7.270	21.140	1436.624

Ipoh) for Greenberg and Greenshields density–speed traffic regression equations.

The other traffic congestion models of Underwood (1961) and Drake et al. (1967) were differed substantially from the Greenberg and Greenshields proposed mathematical methods for traffic congestion. Since the fitted regression equations were unsatisfactory, their estimates of free speed  $v_f$  was extremely high for the Underwood model, and the estimates of  $k_0$  for these traffic congestion techniques were observed to be reasonably similar for the Underwood and Bell-shaped estimates (Tables 6 and 7). However, the estimates on  $v_f$  using Greenshields method were comparable to that of the ball-shaped technique. The authors believe the fitness of estimated regression equations contributed to their differences in the results of the policy variables.

From Table 6, density could be written as follow

$$k = \begin{cases} 7.337 \ln\left(\frac{754.455}{v}\right) & \text{KL-Karak} \\ 40.068 \ln\left(\frac{124.065}{v}\right) & \text{KL-Seremban} \\ 22.005 \ln\left(\frac{227.453}{v}\right) & \text{KL-Ipoh} \end{cases}$$

From Table 7, density could be written as follow

$$k = \begin{cases} 10.734 \left[2 \ln\left(\frac{297.377}{v}\right)\right]^{1/2} & \text{KL-Karak} \\ 56.508 \left[2 \ln\left(\frac{46.554}{v}\right)\right]^{1/2} & \text{KL-Seremban} \\ 31.78836 \left[2 \ln\left(\frac{85.960}{v}\right)\right]^{1/2} & \text{KL-Ipoh} \end{cases}$$

**5.3. Regression model for trans-logarithm density–speed function**

With reference to density–speed relationship the regression equation of trans-log form was fitted to the Malaysia data. The results of these estimated regression equations were obtained and are shown in Table 8. The estimated  $R^2$  values for the trans-log equations for all the three highways were statistically significant at over 80 percent for KL-Karak and KL-Ipoh and around 58 percent for KL-Seremban. For KL-Seremban Highway, the variation in speed could only explained 58 percent of the variations in density. Thus 40 percent of the variations in density were not captured by this density–speed model. In fact all traffic congestion models used in this investigation failed to show sufficient contribution to the density. However, the coefficients of the speed variable and its squared term were highly significant at 0.01 probability level.

**Table 4 – Results of regression analyses between density and speed based on Greenberg model.**

Highway	Density	Constant	$v$ (km/h)	$k_j$ (veh/h)	$v_0$ (km/h)	$R^2$
KL-Karak	$\ln(k)$	3.4687 (45.560)***	-0.0094843 (-7.614)***	32.095	105.437	0.816
KL-Seremban	$\ln(k)$	4.7700 (22.990)***	-0.0240670 (-2.421)**	117.942	41.551	0.592
KL-Ipoh	$\ln(k)$	4.2769 (14.120)***	-0.0171230 (-2.510)**	72.017	58.401	0.832

Note: “\*\*\*” “\*\*” are significant at 0.05 and 0.01 for t-value in parentheses, respectively.

**Table 5 – Results of regression analyses between density and speed based on Greenshields model.**

Highway	Density (veh/h)	Constant	$v$ (km/h)	$k_j$ (veh/h)	$v_f$ (km/h)	$R^2$
KL-Karak	$k$	27.318 (19.740)***	-0.13939 (-5.997)***	27.318	195.982	0.738
KL-Seremban	$k$	111.360 (6.884)***	-1.91020 (-2.464)**	111.360	58.298	0.590
KL-Ipoh	$k$	67.670 (4.557)***	-0.76700 (-2.125)**	67.670	88.227	0.795

Note: “\*\*\*” “\*\*” are significant at 0.05 and 0.01 for t-value in parentheses, respectively.

**Table 6 – Results of regression analyses between density and speed based on Underwood model.**

Highway	Density (veh/h)	Constant	$\ln(v)$	$k_0$ (veh/h)	$v_f$ (km/h)	$R^2$
KL-Karak	$k$	48.619 (6.952)***	-7.3374 (-4.101)***	7.3374	754.455	0.539
KL-Seremban	$k$	193.160 (4.030)***	-40.0680 (-2.524)**	40.0680	124.065	0.597
KL-Ipoh	$k$	119.420 (3.165)***	-22.0050 (-2.053)*	22.0050	227.453	0.791

Note: “\*\*\*” “\*\*” “\*” are significant at 0.1, 0.05 and 0.01 for t-value in parentheses, respectively.

**Table 7 – Results of regression analyses between density and speed based on Drake et al. model.**

Highway	Density	Constant	$v$ (km/h)	$k_0$ (veh/h)	$v_f$ (km/h)	$R^2$
KL-Karak	$k^2$	1312.3 (3.165)***	-230.430 (-2.053)*	10.73400	297.377	0.455
KL-Seremban	$k^2$	24527.0 (3.223)***	-6386.200 (-2.534)**	56.50800	46.554	0.588
KL-Ipoh	$k^2$	9001.3 (2.120)*	-2021.000 (-1.653) <sup>NS</sup>	31.78836	85.960	0.736

Note: “NS” is not significant; “\*\*\*” “\*\*” “\*” are significant at 0.1, 0.05 and 0.01 for t-value in parentheses, respectively.

**Table 8 – Results of regression analyses between density and speed based on trans-log model.**

Highway	Density	$\ln(v)$	$[\ln(v)]^2$	$R^2$	$\ln(v_M)$ [ $v_M$ (km/h)]	$\ln(k_M)$ [ $k_M$ (veh/h)]
KL-Karak	$\ln(k)$	2.0332 (31.850)***	-0.32347 (-20.400)***	0.866	3.143 [23.173]	3.195 [24.410]
KL-Seremban	$\ln(k)$	3.3265 (15.930)***	-0.63243 (-9.200)***	0.579	2.630 [13.875]	4.374 [79.378]
KL-Ipoh	$\ln(k)$	2.7433 (11.280)***	-0.48118 (-7.643)***	0.832	2.851 [17.305]	3.910 [49.900]

Note: “\*\*\*” is significant at 0.01 for t-value in parentheses.

Table 8 also provides the estimates of maximum speed and density both in logarithm and actual values as shown in the last two columns of the table. According to these figures, speed of KL-Karak Highway was somewhat higher 23.173 km/h, therefore exhibited the least congestion level in term of density which was estimated at 24.410 veh/h. The other highways speeds were 13.875 km/h for KL-Seremban and 17.305 km/h for KL-Ipoh. Their densities were estimated at 79.378 veh/h and 49.900 veh/h, respectively.

The potential speed and density both in logarithm and actual values were summarized below given that density was set equal to zero, that is  $\ln(k) = 0$ ,  $v = e^{\alpha/\beta}$ , where  $\alpha$  is the coefficient of speed  $[\ln(v)]$  and  $\beta$  is the coefficient of squared speed  $[\ln(v)]^2$ .

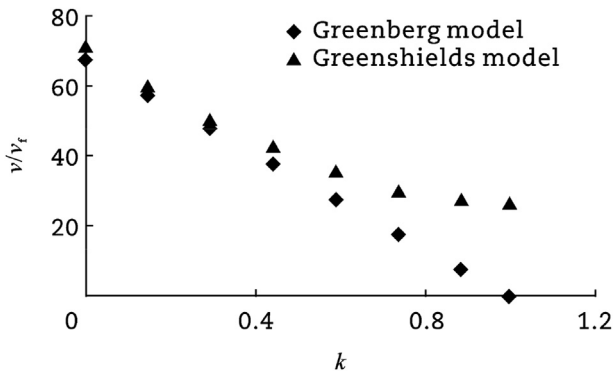
KL-Karak Highway:  $\ln(v) = 6.286$ ,  $v = 536.8$  km/h

KL-Seremban Highway:  $\ln(v) = 5.260$ ,  $v = 192.5$  km/h

KL-Ipoh Highway:  $\ln(v) = 5.701$ ,  $v = 299.2$  km/h

Again the results seemed to support that KL-Karak was less congested in commuting traffic compared to KL-Seremban and KL-Ipoh. The KL-Seremban was most congested of all the three highways analyzed in this investigation.

In general, traffic congestion models are subject to practical applications. The classical models though appear to have explained adequately the relationships between the density and speed using the ratio of congested to the free flow velocity, such as  $v/v_f$  (Greenshields model),  $v/v_0$  (Greenberg model), and the reverse of the ratios ( $v_f/v$ ) (Underwood and Drake et al.



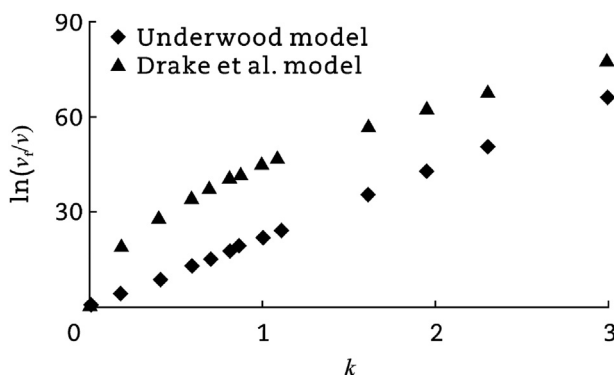
**Fig. 3 – Plot of density–speed using Greenberg and Greenshields models.**

models) actually these classical models are practically limited in their applications.

As illustrated in Figs. 3 and 4, they could depict only one side of the traffic condition such that when  $v = 0$  at the start, velocity is beginning to accelerate. The Greenshields model is linear while the Greenberg model nonlinear. Both of them have the highest point of traffic congestion when velocity is zero. These highest points cannot represent the maximum congestion levels because the classical functions are either linear or exponential. Moreover, all functions fail to yield reasonable results for the second differentiation. The later classical models like Greenberg and Drake et al. seem to have improved the limitation on the linearity assumption of the Greenshields and Underwood. Therefore, the trans-logarithmic function should give a better insight in explaining the traffic congestion phenomena.

## 6. Conclusions

The empirical estimation on flow, density and speed was tested using the trans-logarithm function in view of finding the best fitted steady-state models for selected traffic congestion in Malaysia, namely the KL-Karak Highway, KL-Seremban Highway and KL-Ipoh Highway. Statistical indications showed that the Translog functional form was relevant in explaining



**Fig. 4 – Plot of density–speed using Underwood and Drake et al. models.**

the flow–speed and the density–speed relationships. The economic policy concepts and variables such as the maximum speed or flow and density levels can be derived for identifying, comparing and guidance policy makers in traffic improvisation in view of ever-increasing traffic volume.

The analysis of traffic congestion especially for Malaysia is mainly pioneered and investigated by civil engineers. In this article, the focus is finding out the best form of mathematical models that can appropriately explain and predict the relationships between traffic flow, density and speed as social economist have limited access to such data. Using secondary data collected by the Ministry of Transport Statistics Malaysia 2014 and adjusted for annual growth rate, this information can be applied to approximate the classical traffic congestion models using statistically approved technique of regression analysis. The classical models found most relevant and best fitted in terms of statistical requirements to current data are the Greenberg model and Greenshields model. The other models are also relevant but their coefficients of determination are rather unsatisfactory. Fortunately the trans-logarithm regression equations applied to the current data have significantly and sufficiently fulfilled the requirements of the statistical features.

This work can be of use for policy maker in monitoring traffic congestion if surveyed data are available for further analysis of the Malaysia traffic congestion scenario. Results from survey data of traffic flow, density and speed would supplement the current study.

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