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Conflicting Bifuzzy Multi-attribute Group Decision Making Model with Application to Flood Control Project

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Abstract We propose a group decision making model based on conflicting bifuzzy sets (CBFS) where evaluation are bi-valued in accordance to the subjective assessment obtained from the experts for the positive and negative views. This paper discusses the weighting methods for particular attribute and subattribute with emphasis given to the unification of subjective and objective weights. The integration of CBFS in the model is naturally done by extending the fuzzy evaluation in parallel with the intuitionistic fuzzy. We introduce a new technique to compute the similarity measure, being the degree of agreement between the experts. We end up the paper by demonstrating the applicability of the proposed model to the empirical case of flood control project, one of the project selection problems.

Keywords Multi-attribute group decision making · Conflicting bifuzzy sets · Similarity measure · Fuzzy TOPSIS · Fuzzy AHP

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1 Introduction

Multi-attribute group decision making (MAGDM) is a famous decision making model particularly used in solving the problem of choosing the best candidate from a set of possible alternatives based on evaluation collected from a group of experts. It is admissible that a complex decision problem requires an integration of various expertise in which the lack of knowledge or experience of an expert can be offset by others. Due to its ability in solving the decision making problem with the presence of conflict and agreement among experts, the MAGDM has been successfully applied in constructions (see Cheng and Li 2004), computing and telecommunications technology (see Lee 1996) and energy planning (see Pohekar and Ramachandran 2004).

Evaluation of attribute is naturally vague and ambiguous. A numerical measurement of it may not provides a precise assessment. Thus, the rating of alternative for certain attribute can be better represented using a linguistic approach. Many authors have applied the theory of fuzzy set (FS) introduced by Zadeh (1965) to present attribute in linguistic variable by the means of fuzzy number. For instance, a study by Chen (2000) has demonstrated the ability of fuzzy theory to solve the fuzziness in the technique for order preference by similarity to ideal solution (TOPSIS) procedure. In Tiryaki and Ahlatcioglu (2005), a ranking system has been developed with the use of fuzzy analytic hierarchy process (AHP) in the stock selection problem. A recent paper by Langroudi et al. (2013) used an extended version of fuzzy theory (referred as *type-2 fuzzy set*) for TOPSIS method.

Recently, the intuitionistic fuzzy sets (IFS) founded by Atanassov (1986) has play its role in the multi-attribute decision making processes (see for example the papers by Liu and Wang 2007; Xu and Yager 2008). Ye (2013) has described the weight in the MAGDM under intuitionistic fuzzy setting. The intuitionistic fuzzy provides the membership and non-membership functions, implies that there are two-sided evaluation in IFS. Since the sum of membership and non-membership values must be less or equal to one in IFS, the data are rather restricted. For example, we may rate the candidate in an interview as *good* with membership 0.75 and *bad* with membership 0.25 which is complementary as in FS. The sum of good (membership) and bad (non-membership) may less than one (as in IFS) and may exceed one in some circumstances. We want to study the problem of the sum of two contradict evaluations exceed one by resorting to the so-called conflicting bifuzzy sets (CBFS).

Our aim is to propose a conflicting bifuzzy MAGDM model. A concept paper by Tap (2006) has introduced the conflicting bifuzzy sets and its possible applications, including the decision making problem. Taib et al. (2008) and Zamali et al. (2010) have used the bifuzzy concept in rating alternative for certain criteria in analytic hierarchy process. A study by Xu and Yan (2011) has applied the bifuzzy evaluation to a vendor selection problem. In this paper, we construct the decision matrix for certain attributes and subattributes using the conflicting bifuzzy evaluation. There is no restriction imposed to the evaluation to ensure all data are significant and will be considered in the decision process. This lead us to a fair and better decision.

However, the lack of knowledge and experience of expert (decision maker) may effect the decision process. We weight the expert according to their depth of knowledge and the period of services. In weighting attribute and subattribute, we follow the works done by Liu and Kong (2005) and Wang and Lee (2009) using the integrated fuzzy subjective and objective weights. The fuzzy AHP approach is used to measure the subjective weight, while the objective weight is obtained from the entropy method. The entropy method, referring to Shannon's theory of information (see e.g. Shannon 1949) is one of the objective methods to determine weight (see Zeleny 1982). The advantage of this method is the determination of weight does not require a direct involvement of the decision maker, but a direct function of the information (see Pomerol and Barba-Romero 2000). Some researchers have applied the entropy method to the real world problems. Examples of this literature are Li et al. (2014) and Chen et al. (2014). We integrate the subjective and objective weight using Hurwicz's criterion which reflects the subjective judgment of the decision maker and objective information obtained from the solution of a mathematical model.

We take selection problem of flood control projects as our empirical case. According to Maragoudaki and Tsakiris (2005), most of the previous evaluation and selection of flood control projects are focused mainly on economic and technical factors. For example, in technical aspect, selection of flood control projects are based on the relationship between flood magnitudes (i.e., flood depth, flood velocity, flood flow rate, etc.), in addition to anticipated flood damage. On the other hand, the cost-benefit analysis (CBA) approach is usually used in the economic domain (see Morris-Oswald 2001). The CBA focuses on the implementation and maintenance costs of selected alternative, besides the direct and indirect benefits of total change in income from the project. However, recent approaches in the selection of flood control project recognize the fact that these types of projects interact with various sectors including social, politic, economic and environment. Hence, the application of multi-criteria decision making technique in flood management is clearly significant.

Selection of flood control projects is based on the synthesis of multi-dimensional factors. Therefore, its risk evaluation using a single criteria should be extended to multi-criteria since the routine of single criteria evaluation often omit important information and the integrated risk evaluation for flood control project cannot be obtained. The selection of flood control project also need to consider the social, environment and technical criteria as well as economic criteria in evaluating alternative for sustainable flood management. These problems naturally lead to the use of multi-criteria approach in selection process in which the trade-off between criteria is performed to find out the best solution.

We have collected data of expert's evaluation for the possible alternatives of flood control projects in Kelantan, a state in Malaysia with flood almost once in a year. Malaysia is the country with two seasons in general. The dry season is usually ranging from March to October and rainy season from November to February. With the poor flood management system, the rainy season will be worst to some residents especially farmers. This would be a highly-justified issue since they will lose their income as a result of crop damages from flooding. However, we are not going to investigate the effects of the proposed model to the farmers in this paper. We have consulted three different groups of expert namely *specialize* engineers, local authority and a non-governmental organization (environmentalist). These group of experts will be explained details later.

The rest of the paper is organized as follows. Section 2 discusses the theoretical part consisting the definition of FS and IFS, together with the introduction of CBFS. Later on, we design our MAGDM model with CBFS concept in Sect. 3. Further, we show an application of our proposed model in Sect. 4. Section 5 concludes the paper.

2 Preliminaries

In this section, we state the theoretical parts of fuzzy set and intuitionistic fuzzy sets towards the introduction of conflicting bifuzzy evaluation.

Definition 2.1 Let X be a finite and non-empty set. A fuzzy set A on X is characterized as

$$A = \{< x, \mu_A(x) > | x \in X\}, \tag{2.1}$$

where $\mu_A(x) : X \to [0, 1]$ is the membership function of the fuzzy set A.

In a fuzzy set A, an element x which belongs to a finite set X is given the membership value represents how much x belongs to A. It is clear that fuzzy in all circumstances describing element with a single value.

It is of special interest to have a look at fuzzy number. We will explain a triangular fuzzy number (this should be countered most in this paper) rather than other fuzzy numbers which can be referred in Kaufmann and Gupta (1991). A triangular fuzzy number can be expressed as $A = (a_1, a_2, a_3)$. For each fuzzy number, the membership value is computed using the formula,

$$\mu_A(x) = \begin{Bmatrix} \frac{x-a_1}{a_2-a_1}, & a_1 \le x < a_2 \\ 1, & x = a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 < x \le a_3 \\ 0, & \text{otherwise.} \end{Bmatrix}$$

For $a_1 = a_2 = a_3$, a triangular fuzzy number gives a crisp value and is known as a special case of the fuzzy number.

Definition 2.2 Given two fuzzy numbers $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$, and let λ be any real number. Some operations on fuzzy numbers can be expressed as

$$\begin{aligned} A \times B &= (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3) \\ A/B &= (a_1/b_3, a_2/b_2, a_3/b_1) \\ \lambda \times A &= (\lambda \times a_1, \lambda \times a_2, \lambda \times a_3) \\ A^\lambda &= (a_1^\lambda, a_2^\lambda, a_3^\lambda). \end{aligned} \tag{2.2}$$

We now proceed to the definition of intuitionistic fuzzy set.

Definition 2.3 Let X be a finite and non-empty set. An intuitionistic fuzzy set A in X is expressed as

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}, \tag{2.3}$$

where $\mu_A(x) : X \to [0, 1]$ and $\nu_A(x) : X \to [0, 1]$ are the respective membership and non-membership functions of the fuzzy set A with the condition $0 \leq \mu_A(x)+\nu_A(x) \leq 1$ for all x in X.

As proposed by Atanassov (1986), there exist an intuitionistic index of x in A which can be formulated as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ where $0 \leq \pi_A(x) \leq 1$. It turns out that every fuzzy set A can be represented as the following intuitionistic fuzzy set

$$A = \{< x, \mu_A(x), 1 - \mu_A(x) > |x \in X\},$$

which proves the absence of hesitancy degree in fuzzy set since

$$\pi_A(x) = 1 - \mu_A(x) - (1 - \mu_A(x)) = 0.$$

The $\nu_A(x)$ does not always be $1 - \mu_A(x)$. Therefore, the sum of membership and non-membership degrees can be less than one in IFS. However, if we let those degrees varies within the range $[0, 1]$ for each, then the sum can take any values within the range $[0, 2]$.

Next, we state the definition of conflicting bifuzzy sets retrieved from Tap (2006) (see Zamali et al. 2008).

Definition 2.4 Let X be a finite and non-empty set. If $\left\{A^+, A^-\right\}$ be two fuzzy sets with conflicting characteristic contained in A, then A is called a conflicting bifuzzy set which can precisely defined as

$$A = \{< x, \mu_A(x), \nu_A(x) > |x \in X\}, \tag{2.4}$$

where the function $\mu_A(x) : X \to [0, 1]$ and $\nu_A(x) : X \to [0, 1]$ represent the degree of positivity and degree of negativity of the fuzzy set A respectively for all x in X.

The IFS condition is reformulated to be

$$0 < \mu_A(x) + \nu_A(x) \leq 1 + \varepsilon$$

where ε is a small nonnegative value, $\varepsilon \in \left[0, \frac{1}{2}\right)$. If we simultaneously consider the positivity and negativity, just only one aspect will be dominant at one time, either positive or vice versa. There will be no possible situation where both appears to be dominant. Thus implies that the sum of the degrees cannot exceed $1\frac{1}{2}$.

In order to integrate the positivity and negativity of attribute for a possible alternative, one should have a combination operator to deal with. Such combination operators are the geometric mean (see Gau and Buehrer 1993), arithmetic mean (see Zamali et al. 2008) and multiplicative operator (see Kaufmann and Gupta 1991). For simplicity, we consider arithmetic mean combination operator defined as

$$\phi(\mu_A(x), \eta_A(x)) = \frac{\mu_A(x) + \eta_A(x)}{2}, \tag{2.5}$$

Table 1 Examples of result for different combination operator

Operator	Formula	$(\mu_A(x), \nu_A(x))$		
		(0.7,0.1)	(0.7,0.3)	(0.7,0.5)
Geometric mean	$\phi(\mu_A(x), \eta_A(x)) = \sqrt{\mu_A(x) \times \eta_A(x)}$	0.79	0.70	0.59
Arithmetic mean	$\phi(\mu_A(x), \eta_A(x)) = \frac{\mu_A(x)+\eta_A(x)}{2}$	0.80	0.70	0.60
Multiplicative	$\phi(\mu_A(x), \eta_A(x)) = \mu_A(x) \times \eta_A(x)$	0.63	0.49	0.35

where $\eta_A(x) = 1 - \nu_A(x)$ is the nonnegativity degree. Here, the degree of nonnegativity would rather be a special interest in solving the decision making problem. We show some examples of calculations for different combination operators in Table 1.

3 Conflicting Bifuzzy Multi-attribute Group Decision Making Model

This section presents a group decision making model which demonstrates the applicability of CBFS. We deal with positive and negative aspects concurrently resulting to a *fair* decision. This model involves three stages which generally classified as *rating, aggregation* and *selection*. Figure 1 shows a general framework of the proposed model. At *rating* stage, evaluations of alternatives are given by experts based on the objective and/or subjective attributes. The objective attributes are quantifiable but the subjective are not. Here, the subjective attribute is expressed in lingustic variable and indirectly computed by converting it into the fuzzy number.

The linguistic variables are extensively used to describe characteristic which is not well defined or ambiguous. In the decision making procedure, the rating of alternative itself cannot be expressed by a conventional way, instead we can use linguistic variables such as *good*, *medium* or *poor* to represent it. For our decision making model, we use the linguistic variables described in Table 2 as adopted from Chen and Hwang (1992) and Wang and Lee (2009).

The *aggregation* phase involves setting of weight to each decision maker where the weight is solely depends on how expert is the decision maker to that particular problem. The rating of alternative under weighted subjective attributes at previous stage are then aggregated. For a heterogeneous group of expert, the aggregated decision matrix obtained at this stage will be used for ranking in the *selection* phase. The procedure of fuzzy TOPSIS will be fully utilized in the final stage. The process will last at ranking of alternative based on the value of closeness coefficient. The alternative with the highest closeness coefficient will be chosen as the best one.

3.1 Determination of Weights

3.1.1 Weighting an Expert

We use the simple weighted evaluation technique (WET) (see Olcer and Odabasi 2005; Chiclana et al. 2004) to estimate the weight of the decision makers. Let

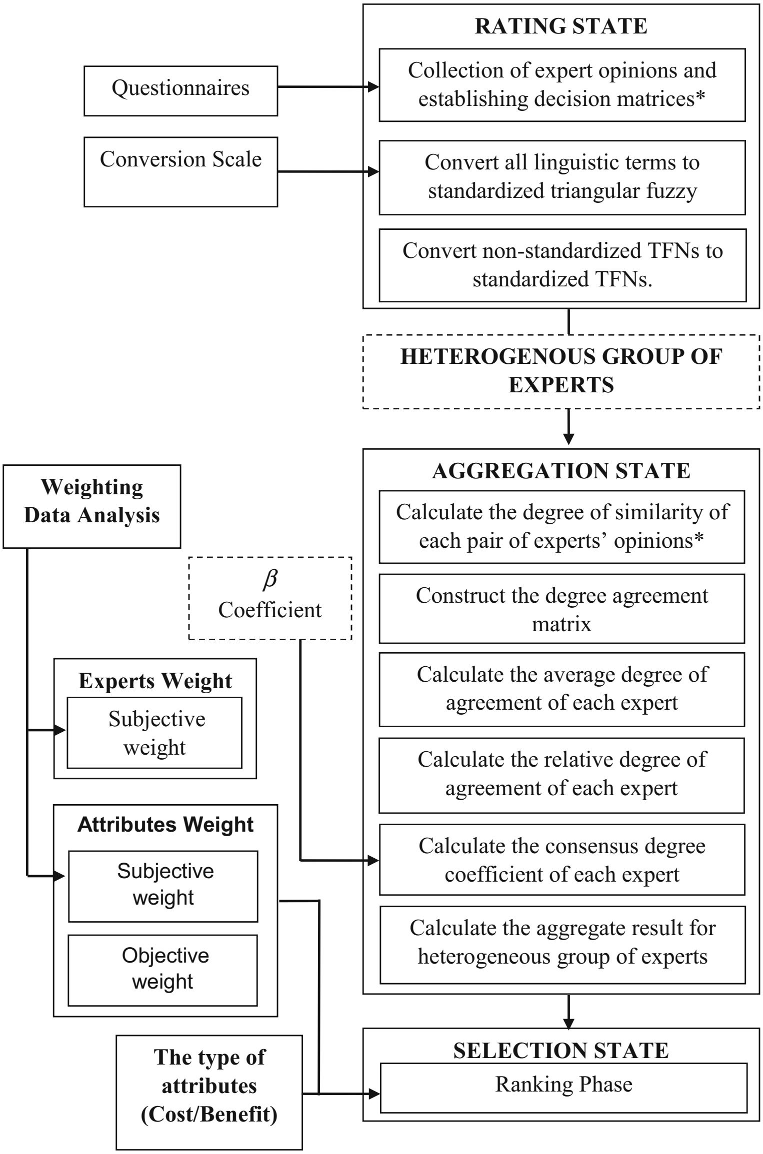

Fig. 1 General framework of the proposed model

$w(e_k)$ be a priority degree of expert $e_k (k = 1, 2, \dots, n)$ where $w(e_k) \in [0, 1]$ and $\sum_{k=1}^{n} w(e_k) = 1$. We first take an expert with the highest priority as proxy and assign value one to him, $r(e_k) = 1$. The relative priority for the expert-l, $r(e_l)(l = 1, 2, \dots, n-1)$ is directly obtained by comparing him to the proxy regarding to his priority in the group of experts. Hence, we have $\max\{r(e_1), r(e_2), \dots, r(e_n)\} = 1$ and $\min\{r(e_1), r(e_2), \dots, r(e_n)\} > 0$. The weight of the decision maker $w(e_k)$ is defined as

Table 2 Linguistic variables for rating alternative

Level of importance	Abbreviation	Fuzzy number
Very Poor/Very Low	VP/VL	(0.00,0.00,0.20)
Poor/Low	P/L	(0.05,0.20,0.35)
Medium Poor/Medium Low	MP/ML	(0.20,0.35,0.50)
Medium	M	(0.35,0.50,0.65)
Medium Good/Medium High	MG/MH	(0.50,0.65,0.80)
Good/High	G/H	(0.65,0.80,0.95)
Very Good/Very High	VG/VH	(0.80,1.00,1.00)

$$w(e_k) = \frac{r(e_k)}{\sum_{k=1}^{n} r(e_k)}. \tag{3.1}$$

If we let the priority of n experts are equal, then $w(e_k) = 1/n$ for $k = 1, 2, \ldots, n$. An example given to precisely demonstrate the weighting method.

Example 3.1 Consider three experts, e_1, e_2 and e_3 are involved. Assume the expert e_1 has absolute knowledge in evaluating an attribute (let say A_1), thus he is assigned as proxy, given the priority $r(e_1) = 1$. Based on how depth is the expertise of other two persons, the priority is given, for instance, $r(e_2) = 0.5$ and $r(e_3) = 0.25$. Using Eq. (3.1), we obtain

$$\begin{aligned} w(e_1) &= 0.571 \\ w(e_2) &= 0.286 \\ w(e_3) &= 0.143 \end{aligned}$$

and $\sum_{k=1}^{3} w(e_k) = 1$.

3.1.2 The Weight of Attribute and Subattribute

In this paper, we integrate the subjective and objective weights to determine the attribute weight which we refer as an integrated weight. The subjective weight reflects the subjective judgment of the decision maker (DM) where the preference information of attributes is directly given, such as AHP method. The objective weight is based on the objective information obtained by solving a mathematical model automatically without any consideration of the decision maker preference. Weight determined by subjective approach reflect the subjective judgment of the decision maker, therefore the ranking of alternatives in fuzzy MAGDM problem contains more arbitrary factors. The objective approach select weight through mathematical calculation, which neglects subjective judgment information of the decision maker (see Liu and Kong 2005).

Determination of weight relies on expert's knowledge and experience which typically characterized as *subjective evaluation* and does not consider the relationship between objects evaluated. By considering AHP as a subjective weight is inadequate

Table 3 Linguistic variable for the weight of attribute and its corresponding fuzzy number

Linguistic term	Mean of fuzzy number	Triangular fuzzy number
Equally important	$\tilde{1}$	(1,1,1)
Intermediate value between $\tilde{1}$ and $\tilde{3}$	$\tilde{2}$	(1,2,3)
Moderately important	$\tilde{3}$	(2,3,4)
Intermediate value between $\tilde{3}$ and $\tilde{5}$	$\tilde{4}$	(3,4,5)
Essentially important	$\tilde{5}$	(4,5,6)
Intermediate value between $\tilde{5}$ and $\tilde{7}$	$\tilde{6}$	(5,6,7)
Very vital important	$\tilde{7}$	(6,7,8)
Intermediate value between $\tilde{7}$ and $\tilde{9}$	$\tilde{8}$	(7,8,9)
Extremely vital important	$\tilde{9}$	(9,9,9)

to capture priority in the assessment of alternative (see Wang et al. 2008). The subjective approach will be more consistent with the integration of objective approach and the integration method is more desirable in the computation of weight. We use the linguistic terms described in Table 3 to form a pairwise comparison matrix where the evaluation is based on its corresponding mean of fuzzy number.

The procedure starts with the determination of weight for attribute-i, w_i^{att}. Assume that a set of m attributes A_i, $i = 1, \ldots, m$ is given. A fuzzy reciprocal judgment matrix for attributes is then defined as

$$D = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1m} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mm} \end{bmatrix},$$

where $\tilde{a}_{ij} = \tilde{1} = (1, 1, 1)$ for all $i = j$ $(i, j = 1, 2, \ldots, m)$ and $\tilde{a}_{ji} = 1/\tilde{a}_{ij}$ for $i \neq j$. By applying the fuzzy synthetic extent, we obtain the corresponding weight for attribute as

$$w_i^{att} = \sum_{i=1, j=1}^{m} \tilde{a}_{ij} \otimes \left[\sum_{i=1}^{m} \sum_{j=1}^{m} \tilde{a}_{ij} \right]^{-1}, \quad i = 1, 2, \ldots, m. \tag{3.2}$$

The weights w_i^{att} are in normalized fuzzy numbers. Note that Eq. (3.2) may result from fuzzy arithmetics, or it can be derived from extension principle.

The attribute A_i normally has k subattributes. Thus, it is important to determine the relative importance of subattribute to that particular attribute. We define the fuzzy judgment matrix for k subattributes with respect to attribute A_i as

$$D_i = \begin{bmatrix} \tilde{a}_{1_i 1_i} & \tilde{a}_{1_i 2_i} & \cdots & \tilde{a}_{1_i k_i} \\ \tilde{a}_{2_i 1_i} & \tilde{a}_{2_i 2_i} & \cdots & \tilde{a}_{2_i k_i} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{k_i 1_i} & \tilde{a}_{k_i 2_i} & \cdots & \tilde{a}_{k_i k_i} \end{bmatrix},$$

where $\tilde{a}_{u_i v_i}$ for $u, v = 1, \ldots, k_i$ is evaluated using Table 3.

By multiplying subattribute's weight to the respective attribute weight in Eq. (3.2), we derive the final weight for subattribute through the aggregation of weights at two consecutive levels as

$$w_{ij}^{agg} = w_i^{att} \otimes w_{ij}^{sub}, \quad \text{for } i, j = 1, 2, \ldots, m, \tag{3.3}$$

where w_{ij}^{agg} is the aggregated fuzzy weight of subattribute and

$$w_{ij}^{sub} = \left[\sum_{i=1}^{k_i} \tilde{a}_{ij} \otimes \left[\sum_{i=1}^{k_i} \sum_{j=1}^{k_i} \tilde{a}_{ij} \right]^{-1} \right].$$

Hence, the entries of the weight vector, w_{ij}^{subj} with length k,

$$\begin{aligned} w_{ij}^{subj} &= \left(w_{ij}^{agg} \right) \\ &= \left(w_{11}^{agg}, w_{12}^{agg}, \ldots, w_{1k_1}^{agg}, w_{21}^{agg}, w_{22}^{agg}, \ldots, w_{2k_2}^{agg}, \ \ldots, w_{m1}^{agg}, w_{m2}^{agg}, \ldots, w_{mk_m}^{agg} \right). \end{aligned}$$

As a part of the procedure in fuzzy AHP method, the determination of consistency index (CI) seems compulsory as it prescribes the acceptance level of the pairwise comparison matrix. To obtain CI, we first multiply the matrix with its priority vector,

$$\begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_m} \\ \frac{w_2}{w_1} & 1 & \cdots & \frac{w_2}{w_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_m}{w_1} & \frac{w_m}{w_2} & \cdots & \frac{w_m}{w_m} \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_m \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \end{bmatrix}.$$

Then, we divide $r_i (i = 1, \ldots, m)$ with its corresponding priority vector,

$$\begin{bmatrix} \frac{r_1}{w_1} \\ \frac{r_2}{w_2} \\ \vdots \\ \frac{r_m}{w_m} \end{bmatrix} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_m \end{bmatrix}.$$

Table 4 Consistency index of a randomly generated reciprocal matrix

m	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

The consistency index can now be computed using

$$\mathrm{CI} = \frac{(\lambda_{\max} - m)}{m - 1}, \tag{3.4}$$

where $\lambda_{\max} = \frac{\lambda_1 + \lambda_2 + \ldots + \lambda_m}{m}$. Finally, we calculate the consistency ratio (CR) using

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}}, \tag{3.5}$$

where RI represents the random index (the consistency index of a randomly generated pairwise comparison matrix). The RI depends on the number of element/criteria m being compared as presented in Table 4. Readers are encouraged to refer Saaty (1980) for detailed of consistency ratio.

We now turn to the procedure of obtaining the objective weight, w_i^{obj}. Suppose we have a decision matrix for n-alternatives and m-attributes, $DM = \left(x_{ij}\right)_{m \times n}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$. By normalizing the DM, we obtain a matrix $\widehat{DM} = \left(z_{ij}\right)$, where z_{ij} is the normalized value of the evaluation of j-th alternative with respect to i-th attribute and $z_{ij} \in [0, 1]$. Among these attributes, to which the bigger is better,

$$z_{ij} = \frac{x_{ij} - \min_j \left\{x_{ij}\right\}}{\max_j \left\{x_{ij}\right\} - \min_j \left\{x_{ij}\right\}}, \tag{3.6}$$

while the smaller is better,

$$z_{ij} = \frac{\max_j \left\{x_{ij}\right\} - x_{ij}}{\max_j \left\{x_{ij}\right\} - \min_j \left\{x_{ij}\right\}}. \tag{3.7}$$

We calculate the entropy using

$$E_{ij} = -k \sum_{i=1}^{m} f_{ij} \ln f_{ij}, \quad i = 1, 2, \ldots, m \tag{3.8}$$

where $f_{ij} = z_{ij} / \sum_{j=1}^{n} z_{ij}$ and $k = 1/\ln n$. The objective weight is defined as

$$w_{ij}^{obj} = \frac{1 - E_{ij}}{m - \sum_{i=1}^{m} E_{ij}}. \tag{3.9}$$

The sum of w_{ij}^{obj} is equal to 1 and $w_{ij}^{obj} \in [0, 1]$.

Lastly, the (subjective) weight calculated from fuzzy AHP is integrated with the (objective) weight to obtain fuzzy integrated weight Ω_{ij} using (see Liu and Kong 2005; Wang et al. 2008),

$$\Omega_{ij} = \frac{\left(w_{ij}^{subj}\right)^{\lambda} \otimes \left(w_{ij}^{obj}\right)^{1-\lambda}}{\sum_{i=1}^{m}\left(\left(w_{ij}^{subj}\right)^{\lambda} \otimes \left(w_{ij}^{obj}\right)^{1-\lambda}\right)}, \tag{3.10}$$

where λ represents the relative importance of the subjective and objective weights to decision maker. Note that the value of subjective weight is in fuzzy number and the objective weight is in crisp. Therefore, the fuzzy integrated weight is a multiplication of fuzzy number with a scalar. The weight is an indicator that does not only shows how important an attribute is, but also shows the level of difference of attribute for different alternatives (see Liu and Kong 2005).

3.2 Rating Phase

Assume that we have n alternatives and m attributes. The CBFS decision matrix is given by

$$DM_{\text{CBFS}} = \begin{bmatrix} (R_{11}^{+}, R_{11}^{-}) & (R_{12}^{+}, R_{12}^{-}) & \cdots & (R_{1j}^{+}, R_{1j}^{-}) \\ (R_{21}^{+}, R_{21}^{-}) & (R_{22}^{+}, R_{22}^{-}) & \cdots & (R_{2j}^{+}, R_{2j}^{-}) \\ \vdots & \vdots & \ddots & \vdots \\ (R_{i1}^{+}, R_{i1}^{-}) & (R_{i2}^{+}, R_{i2}^{-}) & \cdots & (R_{nm}^{+}, R_{nm}^{-}) \end{bmatrix}$$

where $R_{ij}^{+} = \left(a_{ij}^{+}, b_{ij}^{+}, c_{ij}^{+}\right)$ and $R_{ij}^{-} = \left(a_{ij}^{-}, b_{ij}^{-}, c_{ij}^{-}\right)$ are ratings for the positive and negative parts with respect to ith-alternative and jth-attribute being described by a triangular fuzzy number. The rating is based on linguistic variables defined in Table 2.

Some modification should be made to those fuzzy numbers which are not normal. In order to do so, assume that we have a positive triangular fuzzy number $R_{ij}^{*} = (a_{ij}^{*}, b_{ij}^{*}, c_{ij}^{*})$ of rating alternative with respect to subjective attribute where $0 \leq a_{ij}^{(\cdot)} \leq b_{ij}^{(\cdot)} \leq c_{ij}^{(\cdot)} \leq m$. The fuzzy number is converted to a new normalized fuzzy number using

$$R_{ij}^{*} = (a_{ij}^{*}, b_{ij}^{*}, c_{ij}^{*}) = \left(\frac{a_{ij}^{(\cdot)}}{h}, \frac{b_{ij}^{(\cdot)}}{h}, \frac{c_{ij}^{(\cdot)}}{h}\right) \tag{3.11}$$

where $0 \leq a_{ij}^{*} \leq b_{ij}^{*} \leq c_{ij}^{*} \leq 1$ and h is the maximum value of non-normal fuzzy number.

3.3 Aggregation Phase

It is crucial to find a similarity degree for heterogeneous group of experts where different evaluations are given to each alternative. For k finite number of decision makers, we obtain the similarity degree of each pair of expert (e_u, e_v) for $u, v = 1, \ldots, k$ and $u \neq v$ by computing the similarity measure $S_{uv}(R_u, R_v)$. Let $M = R_u = (R_{ij}^+, R_{ij}^-)_u$ and $N = R_v = (R_{ij}^+, R_{ij}^-)_v$, then the similarity measure can be calculated using

$$S_{uv}(M, N) = 1 - \left(\sum_{i=1}^{n} \frac{\varphi_1(x_i) + \varphi_2(x_i)}{2n} \right) \tag{3.12}$$

where,

$$\varphi_1(x_i) = \left| \left(\frac{\mu_M(x_i) + (1 - \gamma_M(x_i))}{2} \right) - \left(\frac{\mu_N(x_i) + (1 - \nu_N(x_i))}{2} \right) \right|, \text{ and}$$

$$\varphi_2(x_i) = \left| \frac{\mu_M(x_i) - \mu_N(x_i)}{2} \right| + \left| \frac{1 - \gamma_M(x_i)}{2} - \frac{1 - \nu_N(x_i)}{2} \right|.$$

The similarity degree measures how similar is M to N for uv-pair of expert. The higher value of $S_{uv}(M, N)$ indicates that M is more similar to N. In other words, if $S_{uv}(M, N) = 1$, then M is almost surely equivalence to N. It is worth noted that $S_{uv}(M, N) = S_{uv}(N, M)$. Further, we construct the agreement matrix as

$$AM = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1k} \\ S_{21} & S_{22} & \cdots & S_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ S_{k1} & S_{k2} & \cdots & S_{kk} \end{bmatrix}$$

where $S_{uv}(M, N) = S_{uv}$ for $u \neq v$ and $S_{uv} = 1$ for $u = v$. We take average of the similarity degree for the expert e_u by computing

$$\bar{S}(e_u) = \frac{1}{k-1} \sum_{\substack{u,v=1 \\ u \neq v}}^{k} S_{uv}. \tag{3.13}$$

Next, we find the relative similarity degree $\tilde{S}(e_u)$ according to (3.13),

$$\tilde{S}(e_u) = \frac{\bar{S}(e_u)}{\sum_{u=1}^{k} \bar{S}(e_u)}. \tag{3.14}$$

By using a relaxation factor β, $(0 \leq \beta \leq 1)$ and the relative similarity degree $\tilde{S}(e_u)$, the consensus coefficient $\widehat{CC}(e_u)$ is calculated as

$$\widehat{CC}(e_u) = \beta w(e_u) + (1 - \beta)\tilde{S}(e_u), \tag{3.15}$$

where $w(e_u)$ is the weight of the expert u obtained from Eq. (3.1). The last step is to compute aggregated fuzzy evaluation, R_{agg} using

$$R_{agg} = \left[\widehat{CC}(e_1) \otimes \hat{R}_1 \oplus \widehat{CC}(e_2) \otimes \hat{R}_2 \oplus \ldots \oplus \widehat{CC}(e_k) \otimes \hat{R}_k\right] \tag{3.16}$$

where

$$\hat{R}_i = \frac{R_i^+ \oplus \left(1 - R_i^-\right)}{2} \tag{3.17}$$

$$= \left(\frac{a_{ij}^+ + \left(1 - c_{ij}^-\right)}{2}, \frac{b_{ij}^+ + \left(1 - b_{ij}^-\right)}{2}, \frac{c_{ij}^+ + \left(1 - a_{ij}^-\right)}{2}\right). \tag{3.18}$$

The aggregated fuzzy evaluation will be used to rank alternatives in the next stage.

3.4 Selection Phase

We present a general idea of fuzzy TOPSIS and the detailed procedure can be referred in Chen (2000). According to *benefit-cost* related attributes, we initially obtain normalized fuzzy decision matrix $\tilde{R} = \left[\tilde{r}_{ij}\right]_{m\times n}$ by normalizing R_{agg} using

$$\tilde{r}_{ij} = \left\{\frac{a_{ij}}{c_{ij}^+}, \frac{b_{ij}}{c_{ij}^+}, \frac{c_{ij}}{c_{ij}^+}\right\}, \qquad c_j^+ = \max c_{ij} \quad \text{if } j \in B,\ i = 1, 2, \ldots, m \tag{3.19}$$

$$\tilde{r}_{ij} = \left\{\frac{a_{ij}^-}{c_{ij}}, \frac{a_{ij}^-}{b_{ij}}, \frac{a_{ij}^-}{a_{ij}}\right\}, \qquad a_j^- = \min a_{ij} \quad \text{if } j \in C,\ i = 1, 2, \ldots, m \tag{3.20}$$

where B and C are the set of benefit criteria and the set of cost criteria respectively.

Next, we calculate the overall performance evaluation of alternative by multiplying the weight to each normalized attribute, $\tilde{v}_{ij} = \tilde{r}_{ij} \otimes \Omega_i$ for $i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, k$, yielding

$$\tilde{V} = \left[\tilde{v}_{ij}\right]_{mk}, \tag{3.21}$$

The positive ideal solution $\tilde{v}^+$ and negative ideal solution $\tilde{v}^-$ will then be computed with $\tilde{v}_i^+ = \max_j \tilde{v}_{ij}$ and $\tilde{v}_i^- = \min_j \tilde{v}_{ij}$ and are sorted in descending order as

$$\tilde{v}^+ = \max\left(\tilde{v}_1^+, \tilde{v}_2^+, \ldots, \tilde{v}_m^+\right), \qquad \tilde{v}^- = \min\left(\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_m^-\right). \tag{3.22}$$

We calculate the distance of the fuzzy decision $\tilde{v}_{ij}$ to the positive ideal solution $\tilde{v}^+ = (a^+, b^+, c^+)$ or negative ideal solution $\tilde{v}^- = (a^-, b^-, c^-)$ using

$$d(\tilde{v}_{ij}, \tilde{v}^{(\cdot)}) = \sqrt{\frac{1}{3}\left[(a_{ij} - a^{(\cdot)})^2 + (b_{ij} - b^{(\cdot)})^2 + (c_{ij} - c^{(\cdot)})^2\right]}, \tag{3.23}$$

and

$$d_i^+ = \sum_{j=1}^{k} d\left(\tilde{v}_{ij}, \tilde{v}^+\right), \qquad d_i^- = \sum_{j=1}^{k} d\left(\tilde{v}_{ij}, \tilde{v}^-\right). \tag{3.24}$$

Finally, the alternatives are ranked by computing its closeness coefficient,

$$\mathrm{CC} = \frac{d_i^-}{d_i^+ + d_i^-}. \tag{3.25}$$

The alternative with the highest closeness coefficient will be chosen as the best alternative.

3.5 Algorithm for CBFS-MAGDM Model

Step 1: Establish a CBFS decision matrix, DM_{CBFS} for each expert.

Step 2: Transform the bifuzzy data into a normalized positive triangular fuzzy number using Eq. (3.11).

Step 3: Assign the relative importance or weight for experts and attributes using Eq. (3.1)–(3.10).

Step 4: Measure the similarity degree using Eq. (3.12). Construct the agreement matrix, the average degree of agreement, the relative degree of agreement and the consensus coefficient by using Eq. (3.13)–(3.15). Then, aggregate all expert's fuzzy evaluation for each alternative using Eq. (3.16).

Step 5: Construct the normalized rating and weighted normalized rating using Eq. (3.19)–(3.21).

Step 6: Calculate the positive-ideal solution, the negative-ideal solution and compute the distance of fuzzy decision to the positive and negative ideal solution using Eq. (3.22)–(3.24).

Step 7: Calculate the closeness coefficient CC using Eq. (3.25). Rank the alternative according to the value of its closeness coefficient.

4 On the Selection of Flood Control Project

We applied our model to the selection problem of flood control project. There are four alternatives to be considered namely reservoir (X_1), channel improvement (X_2), diversion scheme (X_3) and dikes (X_4). Each alternative is evaluated based on four attributes consist of economical factor (A_1), social factor (A_2), environmental factor (A_3) and technical factor (A_4). The attributes together with their corresponding subattributes are listed in Table 5. Evaluation of alternatives with respect to attribute and subattribute are bi-valued except for money term factors, A_{11} and A_{12} and a time-based factor, A_{41}. We choose three group of experts in the decision processes including *specialize* engineers in the Department of Drainage and Irrigation, Kelantan (e_1), Kelantan's local authority (e_2) and Malaysian non-governmental organization (e_3).

First, we establish a CBFS decision matrix for *rating* alternative with respect to the given attributes and subattributes. We refer to Step 1 in Sect. 3.5. The rating of alternatives are obtained from expert's evaluation and presented in Table 6. We see that the ratings are in linguistic variable being described in Table 2, except evaluation for project cost (A_{11}), operation and maintenance (A_{12}) and lifetime (A_{41}). For X_1,

Table 5 List of attributes and corresponding subattributes

Attribute	Subattribute
Economical (A_1)	Project cost (A_{11})
	Operation and maintenance cost (A_{12})
	Project benefit (A_{13})
	Reliability economic parameter (A_{14})
Social (A_2)	Social acceptability (A_{21})
	Effect on demographic (A_{22})
	Effect on structure (A_{23})
	Recreation activity (A_{24})
Environmental (A_3)	Water quality impact (A_{31})
	Nature conservation (A_{32})
	Soil impact (A_{33})
	Landscape (A_{34})
	Sanitary condition (A_{35})
Technical (A_4)	Lifetime (A_{41})
	Adaptability (A_{42})
	Level of protection (A_{43})
	Technical complexity (A_{44})
	Flexibility (A_{45})

the expert e_2 and e_3 rated *medium high* to the positive part of soil impact (A_{33}), but the rating for negative side are different. Expert e_2 felt that reservoir will result of *moderate low* negative impact to soil and e_3 thought of *low* negative soil impact.

The experts agreed that the cost of running all alternatives is very high, approximately ranging from 0.6 to 2.1 billion Ringgit Malaysia (reflects the cost for maintenance and project and operation). Furthermore, different alternative has different lifetime. Reservoir can retain up to 100 years while the rest can operates just in the 10 years. At first glimpse, we see that reservoir is not efficient economically as it requires very high running cost eventhough the contribution to the positive impact to social and environment is high. The channel improvement is the most admissable if the budget is limited, but the positive impact to all factors are considerably moderate. The other two alternatives have moderate impact to all factors.

Some subattributes in the decision matrix are in money term and time-based which should be normalized. We refer to Step 2 in Sect. 3.5 to normalize rating for subattributes A_{11}, A_{12} and A_{41}. While other factor remains the same, we just show the normalized decision matrix for those three subattributes in Table 7. Due to the lack of knowledge of expert for certain attribute, we define weight to each expert as in Step 3 in Sect. 3.5. We admit that *specialize* engineer, e_1 has the highest priority in this decision making processes. Therefore, we choose e_1 to be the proxy and the weight of experts will be determined according to the degree of importance as calculated using Eq. (3.1). In parallel, we calculate the weight for attribute (and subattribute) using fuzzy AHP for subjective weight and entropy method for objective weight. The results for subjective and objective weights are shown in Tables 8 and 9 respectively.

Table 6 The decision matrix of rating alternatives

Attribute	Subattribute	X_1			X_2			X_3			X_4		
		e_1	e_2	e_3	e_1	e_2	e_3	e_1	e_2	e_3	e_1	e_2	e_3
A_1	A_{11}	2			1			0.5			0.5		
	A_{12}	0.1			0.5			0.1			0.5		
	A_{13}	(VH,L)	(VH,L)	(VH,L)	(M,M)	(MH,ML)	(M,M)	(MH,ML)	(H,ML)	(H,ML)	(H,L)	(MH,L)	(H,L)
	A_{14}	(H,L)	(MH,L)	(MH,ML)	(M,ML)	(M,ML)	(M,ML)	(MH,ML)	(M,ML)	(H,ML)	(H,L)	(MH,L)	(MH,L)
A_2	A_{21}	(H,L)	(MH,ML)	(M,L)	(VH,VL)	(MH,L)	(MG,L)	(MH,ML)	(H,ML)	(MH,ML)	(MH,L)	(MH,L)	(MH,L)
	A_{22}	(L,M)	(H,ML)	(MH,ML)	(H,VL)	(H,L)	(H,L)	(M,M)	(MH,M)	(MH,M)	(MH,ML)	(H,ML)	(H,ML)
	A_{23}	(H,L)	(MH,M)	(H,ML)	(M,VL)	(M,L)	(M,L)	(M,L)	(MH,ML)	(M,ML)	(MH,L) (H,L)	(H,L)	
	A_{24}	(VH,L)	(H,L)	(H,L)	(VL,M)	(L,M)	(VL,M)	(VH,ML)	(H,ML)	(H,ML)	(VH,L)	(MH,L)	(MH,L)
A_3	A_{31}	(H,L)	(MH,L)	(MH,L)	(M,M)	(MH,ML)	(H,M)	(M,L)	(MH,L)	(M,L)	(MH,L)	(MH,L)	(H,L)
	A_{32}	(MH,L)	(M,ML)	(M,L)	(M,M)	(L,H)	(VL,H)	(ML,M)	(M,ML)	(ML,M)	(M,L)	(M,ML)	(M,L)
	A_{33}	(MH,L)	(MH,ML)	(MH,L)	(M,M)	(M,M)	(ML,MH)	(MH,L)	(M,ML)	(MH,M)	(M,L)	(M,L)	(M,ML)
	A_{34}	(VH,L)	(VH,L)	(H,L)	(VL,H)	(L,M)	(L,MH)	(VH,L)	(H,L)	(H,L)	(VH,L)	(H,VL)	(H,L)
	A_{35}	(MH,L)	(MH,ML)	(H,ML)	(VL,H)	(L,H)	(VL,H)	(MH,ML)	(M,L)	(MH,ML)	(MH,L)	(M,L)	(MH,ML)
A_4	A_{41}	100 year			10 year			10 year			10 year		
	A_{42}	(VH,L)	(H,ML)	(H,L)	(M,M)	(M,ML)	(M,ML)	(MH,ML)	(H,ML)	(H,ML)	(H,L)	(MH,L)	(MH,ML)
	A_{43}	(VH,L)	(H,ML)	(H,L)	(ML,H)	(M,L)	(ML,MH)	(MH,ML)	(H,L)	(H,L)	(H,L)	(MH,L)	(H,L)
	A_{44}	(H,L)	(H,L)	(MH,L)	(M,ML)	(MH,L)	(M,ML)	(M,MH)	(MH,M)	(H,M)	(H,L)	(MH,ML)	(MH,ML)
	A_{45}	(VH,L)	(H,L)	(VH,L)	(M,M)	(M,ML)	(M,ML)	(H,ML)	(MH,L)	(H,ML)	(MH,L)	(M,L)	(M,L)

Table 7 The normalized decision matrix for the three subattributes

Attribute	X_1	X_2	X_3	X_4
A_{11}	(0.818,0.909,1.000)	(0.364,0.445,0.545)	(0.136,0.227,0.318)	(0.136,0.227,0.318)
A_{12}	(0.091,0.182,0.273)	(0.818,0.909,1.000)	(0.091,0.182,0.273)	(0.818,0.909,1.000)
A_{41}	(0.091,0.182,0.273)	(0.073,0.091,0.109)	(0.073,0.091,0.109)	(0.073,0.091,0.109)

Table 8 The subjective weight for attributes and subattributes and the aggregated weight

Attribute	w_i^{att}	Subattribute	w_{ij}^{sub}	w_{ij}^{agg}
A_1	(0.304,0.460,0.687)	A_{11}	(0.140,0.239,0.391)	(0.042,0.110,0.269)
		A_{12}	(0.147,0.220,0.343)	(0.045,0.101,0.236)
		A_{13}	(0.319,0.489,0.742)	(0.097,0.225,0.509)
		A_{14}	(0.040,0.052,0.072)	(0.012,0.024,0.049)
A_2	(0.237,0.353,0.534)	A_{21}	(0.256,0.471,0.840)	(0.061,0.166,0.449)
		A_{22}	(0.083,0.164,0.330)	(0.020,0.058,0.176)
		A_{23}	(0.139,0.278,0.540)	(0.033,0.098,0.288)
		A_{24}	(0.056,0.087,0.165)	(0.013,0.031,0.088)
A_3	(0.085,0.137,0.213)	A_{31}	(0.178,0.334,0.637)	(0.015,0.047,0.136)
		A_{32}	(0.076,0.137,0.262)	(0.006,0.019,0.056)
		A_{33}	(0.145,0.278,0.524)	(0.012,0.038,0.112)
		A_{34}	(0.037,0.056,0.097)	(0.003,0.008,0.021)
		A_{35}	(0.092,0.185,0.375)	(0.008,0.025,0.080)
A_4	(0.038,0.050,0.071)	A_{41}	(0.185,0.293,0.458)	(0.007,0.015,0.033)
		A_{42}	(0.142,0.225,0.366)	(0.005,0.011,0.026)
		A_{43}	(0.230,0.367,0.576)	(0.009,0.019,0.041)
		A_{44}	(0.042,0.075,0.126)	(0.002,0.004,0.009)
		A_{45}	(0.029,0.041,0.068)	(0.001,0.002,0.005)

For the sake of simplicity, we use the mean of fuzzy number as in Table 3 to calculate the consistency ratio. We obtain the consistency ratio equal 0.059 accepting the validity of our pairwise comparison matrix.

Expert's rating are then aggregated using Eq. (3.12)–(3.16). We refer to Step 4 in Sect. 3.5. The value β is set to 0.4 represent the expert dominance for this problem. We report the aggregated fuzzy rating in Table 10. Next, the normalized ratings and weighted normalized ratings of the matrices are constructed using Eq. (3.19)–(3.21). We refer to Step 5 in Sect. 3.5. Tables 11 and 12 respectively present the fuzzy normalized rating and the weighted fuzzy normalized rating. Note that Table 12 is obtained by multiplying the integrated weight reported in Table 9 with fuzzy normalized rating.

The positive ideal solution and negative ideal solution are then calculated using Eq. (3.23), and the distance measure is computed using Eq. (3.25). We refer to Step 6 in Sect. 3.5. We simply determine the positive ideal solution by taking the element with the highest value for the benefit attribute and the element with the lowest value

Table 9 The objective weight and integrated weight

Attribute	Subattribute	Entropy	w_{ij}^{obj}	Ω_{ij}
A_1	A_{11}	0.781	0.040	(0.0017,0.0044,0.0108)
	A_{12}	0.500	0.091	(0.0041,0.0092,0.0215)
	A_{13}	0.753	0.045	(0.0044,0.0101,0.0229)
	A_{14}	0.763	0.043	(0.0005,0.0010,0.0021)
A_2	A_{21}	0.646	0.064	(0.0039,0.0106,0.0287)
	A_{22}	0.588	0.075	(0.0015,0.0043,0.0132)
	A_{23}	0.728	0.049	(0.0016,0.0048,0.0141)
	A_{24}	0.791	0.038	(0.0005,0.0012,0.0033)
A_3	A_{31}	0.762	0.043	(0.0006,0.0020,0.0058)
	A_{32}	0.766	0.042	(0.0003,0.0008,0.0024)
	A_{33}	0.785	0.039	(0.0005,0.0015,0.0044)
	A_{34}	0.792	0.038	(0.0001,0.0003,0.0008)
	A_{35}	0.792	0.038	(0.0003,0.0010,0.0030)
A_4	A_{41}	0.000	0.181	(0.0013,0.0027,0.0060)
	A_{42}	0.773	0.041	(0.0002,0.0005,0.0011)
	A_{43}	0.790	0.038	(0.0003,0.0007,0.0016)
	A_{44}	0.726	0.050	(0.0001,0.0002,0.0005)
	A_{45}	0.741	0.047	(0.0000,0.0001,0.0002)

Table 10 The aggregated fuzzy rating for heterogeneous group of experts

Attribute	Subattribute	X_1	X_2	X_3	X_4
A_1	A_{11}	(0.818,0.909,1.000)	(0.364,0.455,0.545)	(0.136,0.227,0.318)	(0.136,0.227,0.318)
	A_{12}	(0.091,0.182,0.273)	(0.818,0.909,1.000)	(0.091,0.182,0.273)	(0.818,0.909,1.000)
	A_{13}	(0.725,0.900,0.975)	(0.408,0.558,0.708)	(0.548,0.698,0.848)	(0.687,0.770,0.920)
	A_{14}	(0.587,0.737,0.887)	(0.425,0.575,0.725)	(0.487,0.637,0.787)	(0.603,0.753,0.903)
A_2	A_{21}	(0.572,0.722,0.872)	(0.652,0.820,0.918)	(0.530,0.680,0.830)	(0.575,0.725,0.875)
	A_{22}	(0.436,0.586,0.736)	(0.695,0.860,0.965)	(0.400,0.550,0.700)	(0.550,0.700,0.850)
	A_{23}	(0.558,0.708,0.858)	(0.524,0.682,0.808)	(0.470,0.620,0.770)	(0.626,0.776,0.926)
	A_{24}	(0.674,0.832,0.958)	(0.192,0.318,0.476)	(0.559,0.757,0.883)	(0.621,0.779,0.906)
A_3	A_{31}	(0.602,0.752,0.902)	(0.446,0.596,0.746)	(0.518,0.668,0.818)	(0.605,0.755,0.905)
	A_{32}	(0.523,0.673,0.823)	(0.123,0.242,0.402)	(0.318,0.468,0.618)	(0.478,0.628,0.778)
	A_{33}	(0.557,0.707,0.857)	(0.408,0.558,0.708)	(0.509,0.659,0.809)	(0.500,0.065,0.800)
	A_{34}	(0.701,0.868,0.967)	(0.529,0.686,0.816)	(0.641,0.798,0.927)	(0.725,0.900,0.975)
	A_{35}	(0.552,0.702,0.852)	(0.032,0.130,0.297)	(0.500,0.650,0.800)	(0.530,0.680,0.830)
A_4	A_{41}	(0.818,0.909,1.000)	(0.073,0.091,0.109)	(0.073,0.091,0.109)	(0.073,0.091,0.109)
	A_{42}	(0.680,0.840,0.960)	(0.395,0.545,0.695)	(0.545,0.695,0.845)	(0.605,0.755,0.905)
	A_{43}	(0.656,0.816,0.936)	(0.259,0.409,0.559)	(0.592,0.742,0.892)	(0.623,0.776,0.926)
	A_{44}	(0.632,0.782,0.932)	(0.477,0.627,0.777)	(0.384,0.534,0.684)	(0.558,0.708,0.858)
	A_{45}	(0.698,0.864,0.966)	(0.395,0.545,0.695)	(0.575,0.725,0.875)	(0.315,0.680,0.830)

Table 11 Fuzzy normalized ratings for heterogeneous group of experts

Attribute	Subattribute	X_1	X_2	X_3	X_4
A_1	A_{11}	(0.136,0.150,0.166)	(0.249,0.299,0.374)	(0.427,0.598,1.000)	(0.427,0.598,1.000)
	A_{12}	(0.333,0.500,1.000)	(0.091,0.100,0.111)	(0.333,0.500,1.000)	(0.091,0.100,0.111)
	A_{13}	(0.744,0.923,1.000)	(0.419,0.573,0.726)	(0.562,0.716,0.870)	(0.705,0.790,0.944)
	A_{14}	(0.650,0.816,0.982)	(0.471,0.637,0.803)	(0.539,0.705,0.871)	(0.668,0.834,1.000)
A_2	A_{21}	(0.623,0.787,0.950)	(0.711,0.893,1.000)	(0.577,0.740,0.904)	(0.626,0.790,0.953)
	A_{22}	(0.451,0.607,0.762)	(0.720,0.892,1.000)	(0.414,0.570,0.725)	(0.570,0.725,0.881)
	A_{23}	(0.603,0.765,0.927)	(0.565,0.736,0.872)	(0.507,0.669,0.831)	(0.676,0.838,1.000)
	A_{24}	(0.703,0.868,1.000)	(0.201,0.332,0.497)	(0.625,0.790,0.922)	(0.649,0.813,0.946)
A_3	A_{31}	(0.665,0.831,0.997)	(0.493,0.659,0.825)	(0.572,0.738,0.904)	(0.668,0.834,1.000)
	A_{32}	(0.635,0.818,1.000)	(0.149,0.294,0.489)	(0.387,0.569,0.751)	(0.581,0.763,0.945)
	A_{33}	(0.650,0.825,1.000)	(0.476,0.651,0.826)	(0.594,0.769,0.944)	(0.583,0.758,0.933)
	A_{34}	(0.719,0.891,0.992)	(0.543,0.704,0.837)	(0.658,0.818,0.951)	(0.744,0.923,1.000)
	A_{35}	(0.648,0.824,1.000)	(0.038,0.152,0.349)	(0.587,0.763,0.939)	(0.622,0.799,0.975)
A_4	A_{41}	(0.818,0.909,1.000)	(0.073,0.091,0.109)	(0.073,0.091,0.109)	(0.073,0.091,0.109)
	A_{42}	(0.708,0.875,1.000)	(0.412,0.568,0.724)	(0.568,0.724,0.881)	(0.631,0.787,0.943)
	A_{43}	(0.701,0.872,1.000)	(0.276,0.436,0.597)	(0.632,0.792,0.952)	(0.666,0.829,0.990)
	A_{44}	(0.678,0.839,1.000)	(0.512,0.673,0.834)	(0.412,0.573,0.734)	(0.599,0.760,0.921)
	A_{45}	(0.723,0.895,1.000)	(0.409,0.565,0.720)	(0.595,0.750,0.906)	(0.326,0.703,0.859)

Table 12 The weighted fuzzy normalized ratings for heterogeneous group of experts

Attribute	Subattribute	X_1	X_2	X_3	X_4
A_1	A_{11}	(0.004,0.012,0.032)	(0.007,0.023,0.072)	(0.013,0.047,0.193)	(0.013,0.047,0.193)
	A_{12}	(0.016,0.056,0.274)	(0.004,0.011,0.030)	(0.016,0.056,0.274)	(0.004,0.011,0.030)
	A_{13}	(0.036,0.109,0.282)	(0.021,0.070,0.210)	(0.027,0.084,0.245)	(0.039,0.092,0.264)
	A_{14}	(0.011,0.031,0.086)	(0.008,0.024,0.069)	(0.009,0.026,0.073)	(0.011,0.031,0.086)
A_2	A_{21}	(0.028,0.094,0.298)	(0.033,0.110,0.317)	(0.026,0.090,0.287)	(0.028,0.095,0.300)
	A_{22}	(0.013,0.047,0.163)	(0.020,0.068,0.215)	(0.012,0.044,0.156)	(0.016,0.056,0.189)
	A_{23}	(0.018,0.063,0.209)	(0.017,0.060,0.194)	(0.015,0.054,0.184)	(0.020,0.069,0.223)
	A_{24}	(0.012,0.035,0.108)	(0.003,0.013,0.054)	(0.010,0.032,0.100)	(0.011,0.033,0.102)
A_3	A_{31}	(0.012,0.044,0.142)	(0.009,0.034,0.116)	(0.010,0.038,0.127)	(0.012,0.044,0.143)
	A_{32}	(0.008,0.027,0.091)	(0.002,0.009,0.043)	(0.005,0.019,0.068)	(0.007,0.026,0.087)
	A_{33}	(0.011,0.037,0.124)	(0.008,0.030,0.103)	(0.010,0.035,0.117)	(0.009,0.034,0.114)
	A_{34}	(0.006,0.018,0.052)	(0.004,0.014,0.043)	(0.005,0.016,0.049)	(0.006,0.018,0.052)
	A_{35}	(0.008,0.030,0.103)	(0.000,0.005,0.035)	(0.007,0.028,0.096)	(0.008,0.029,0.100)
A_4	A_{41}	(0.021,0.055,0.144)	(0.002,0.006,0.016)	(0.002,0.006,0.016)	(0.002,0.006,0.016)
	A_{42}	(0.008,0.022,0.061)	(0.004,0.014,0.044)	(0.006,0.018,0.054)	(0.007,0.020,0.058)
	A_{43}	(0.009,0.026,0.074)	(0.004,0.013,0.043)	(0.008,0.024,0.069)	(0.009,0.026,0.073)
	A_{44}	(0.004,0.014,0.040)	(0.003,0.011,0.033)	(0.002,0.009,0.028)	(0.004,0.012,0.037)
	A_{45}	(0.004,0.010,0.028)	(0.002,0.006,0.020)	(0.003,0.009,0.026)	(0.002,0.008,0.025)

Table 13 The positive ideal solution and negative ideal solution for heterogeneous group of experts

Attribute	Subattribute	$\tilde{v}^+$	$\tilde{v}^-$
A_1	A_{11}	(0.013,0.047,0.193)	(0.004,0.012,0.032)
	A_{12}	(0.016,0.056,0.274)	(0.004,0.011,0.030)
	A_{13}	(0.039,0.109,0.282)	(0.021,0.070,0.210)
	A_{14}	(0.011,0.031,0.086)	(0.008,0.024,0.069)
A_2	A_{21}	(0.033,0.110,0.317)	(0.026,0.090,0.287)
	A_{22}	(0.020,0.068,0.215)	(0.012,0.044,0.156)
	A_{23}	(0.020,0.069,0.223)	(0.015,0.054,0.184)
	A_{24}	(0.012,0.035,0.108)	(0.003,0.013,0.054)
A_3	A_{31}	(0.012,0.044,0.143)	(0.009,0.034,0.116)
	A_{32}	(0.008,0.027,0.091)	(0.002,0.009,0.043)
	A_{33}	(0.011,0.037,0.124)	(0.008,0.030,0.103)
	A_{34}	(0.006,0.018,0.052)	(0.004,0.014,0.043)
	A_{35}	(0.008,0.030,0.103)	(0.000,0.005,0.035)
A_3	A_{41}	(0.021,0.055,0.144)	(0.002,0.006,0.016)
	A_{42}	(0.008,0.022,0.061)	(0.004,0.014,0.044)
	A_{43}	(0.009,0.027,0.074)	(0.004,0.013,0.043)
	A_{44}	(0.004,0.014,0.040)	(0.002,0.009,0.028)
	A_{45}	(0.004,0.010,0.028)	(0.002,0.006,0.020)

Table 14 Distance measure to positive ideal solution and negative ideal solution using CBFS-MAGDM

	X_1	X_2	X_3	X_4
d_i^+	0.152	0.552	0.249	0.286
d_i^-	0.492	0.092	0.395	0.359
CC	0.764	0.142	0.613	0.556
Ranking	1	4	2	3

for the cost attribute. In contrast, the negative ideal solution is determined by taking the element with opposite values of benefit and cost attributes. The result are reported in Table 13. Next, Table 14 shows the distance measure to the positive and negative ideal solutions and its corresponding closeness coefficient as in Step 7 in Sect. 3.5.

We found $X_1 \succ X_3 \succ X_4 \succ X_2$, meaning that the best flood control project is reservoir and the worst is channel improvement. Reservoir is the highest cost project which was initially seems unefficient as it budget sensitive. However, it has a longer lifetime and a very high rating for its technicality on average. Furthermore, it conserves nature and has a high impact to society. Eventhough channel improvement use less money but it should be maintened for estimated every 10 years. In addition, we see that it has on average moderate impact to social and environment. Thus, the experts have selected reservoir to be the best alternative in controlling the flood.

In comparison, we show the ranking calculated based on fuzzy MAGDM in Table 15. We see that the ranking are similar for fuzzy MAGDM and CBFS-MAGDM. In general, the CC value for all alternatives are greater for CBFS-MAGDM except for channel improvement (X_2) where the earlier method gives CC's value 0.300. The

Table 15 Distance measure to the positive ideal solution and negative ideal solution using fuzzy MAGDM

	X_1	X_2	X_3	X_4
d_i^+	0.313	0.666	0.441	0.470
d_i^-	0.638	0.285	0.511	0.482
CC	0.671	0.300	0.537	0.506
Ranking	1	4	2	3

Table 16 Estimated CC for different β

β	X_1	X_2	X_3	X_4
0.0	0.765	0.141	0.621	0.558
0.2	0.764	0.142	0.617	0.557
0.4	0.764	0.142	0.613	0.556
0.6	0.764	0.143	0.610	0.556
0.8	0.763	0.143	0.607	0.555
1.0	0.761	0.144	0.603	0.554

result justifies the influence of two-sided judgment. In the case of complementary, i.e the sum of positive and negative membership values equal one, the decision process may not influenced by any of evaluation's side. Therefore, fuzzy approach is adequate. For no complementary case, the result may change. If the sum of positive and negative evaluation is less than one, we will see a greater result. On the other hand, if the sum of positive and negative aspects is greater than one, then one will have a lower result.

Another important note is the objective weight for fuzzy MAGDM derived from entropy method is different from the case of CBFS-MAGDM since the analysis data is based on single-side evaluation of experts. Hence, the final weight (an integration of the subjective and objective weights) of CBFS-MAGDM method also produces different results and affects the final decision.

4.1 Sensitivity of Coefficient β

Sensitivity analysis is performed to see the effect of coefficient β to the final ranking. The β takes value in the interval [0, 1]. Table 16 shows the closeness coefficient computed for several β. Since the CC values do not much deviate, we may conclude that this case is not β sensitive. The ranking stays at relatively the same level.

5 Conclusion

We have designed a group decision making model based on a new conflicting bifuzzy approach. The new model considers two conflicting perspectives, i.e. positive and negative views which will then be integrated using arithmetic mean combination operator. We show that the model is fair enough to deal with the (not significant) data restricted by FS and IFS. Thus, the proposed model provides a better solution to the empirical case with similar features.

The weight is given to the decision makers, attributes and subattributes represent their degree of importance. For the decision maker, the weight is calculated using weighted evaluation technique. We integrate the subjective and objective weights for the attribute and subattribute which reflects the subjective rating of the expert and objective information obtained from a mathematical model respectively. The fuzzy AHP method computes the subjective weight, and we use the entropy method to calculate the objective weight.

We show the capability of the model using data of expert's rating for flood control project. The pairwise comparison matrix was performed to rate the alternatives. By having consistency ratio in the fuzzy AHP method, we see that the rating matrix is acceptable. To aggregate the rating, we present several different expert's dominance for the decision problem. The result indicates that the dominance issue is not so important in the decision process.

In comparison to the fuzzy MAGDM model, we show the final result of both methods. The fuzzy MAGDM and CBFS-MAGDM models produce similar ranking; the reservoir is the best solution and the channel improvement is the last. However, the value of the closeness coefficient is different. We see that CBFS-MAGDM gives higher closeness coefficient.

We suggest to formulate conflicting index in the bifuzzy evaluation for further research. The conflicting index can be defined as the degree of conflict between positive and negative views to certain attribute. One can use our suggested similarity measure of two experts as a stepping stone but this is not the case of conflicting between experts, but would be an internal conflict of attribute which can be measured externally. Also, the objective weight can be alternatively measured using a reference point method (see e.g. Wierzbicki 1980; Wierzbicki et al. 2000), ordered weighted average, choquet integral weight which are left for future extension.

The application of our proposed model is not limited to the selection problem of flood control projects. We may apply the model to other real decision problems having similar features, namely the: conflicting condition, incomplete information and non-standard data structure. The potential area are in construction project and energy planning. Our model is designed to solve the conflicting judgment between the experts, thus provides a better policy for the use of energy in the future. The construction project is similar to our empirical case. With the aim to minimizing the cost and maximizing the profit, the conflicting condition and incomplete information for each candidate should be carefully revised. However, this model is rich of complexity, thus stipulate one to consume expensive time to run.

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