

## THE PERFORMANCE OF BB-MCEWMA MODEL: CASE STUDY ON SUKUK RANTAU ABANG CAPITAL BERHAD, MALAYSIA

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**Abstract:** Monitoring process of auto-correlated data has become an impressive impact in Statistical Process Control (SPC). It is due to the nature sensitivity where the current information data is said to be correlate to previous data, and well as the data can be categorised as dependent data. Thus, this study considered an alternative chart and particularly known as MCEWMA chart. The function is to quick detection on small shifts in the monitoring process. However, monitoring the autocorrelation having an indirect problem where it is often influenced by inaccurate estimation from its base model. Ignoring this problem will eventually cause another problem such as low performance of model where it said to be less effective. Thus, to solve this problem, a double bootstrap approach is hybrid into the MCEWMA chart. The main objective of this study is to construct a standard error of confidence intervals of MCEWMA. The algorithm in this study eventually comes out with a new model coins as double bootstrap MCEWMA (BB-MCEWMA). To see the effectiveness the proposed model, real data will implemented, which sukuk (Islamic financial instrument) is issued by Rantau Abang Capital Berhad, Malaysia. The trade date for the sukuk is taken from March 2006 until March 2011. The performance of BB-MCEWMA in terms of effectiveness of point estimator and interval estimator (Normal, Student's-t and BCa) for the proposed model and the MCEWMA are considered. BB-MCEWMA gives smallest value of error (MSE and RMSE) and shortest length of interval estimation. With these result, it is proven that the hybrid of double bootstrap approach into MCEWMA model fixed the estimation of original model and single bootstrap approach model, and statistically give more accurate estimation model.

**Keyword:** Double Bootstrap, sukuk, point estimator, interval estimator

### 1. INTRODUCTION

In Statistical Process Control (SPC), model estimation of chart depends on characteristic of sample data used. Before the estimation of model and monitoring process, the characteristic of interest sample is analysis to determine whether the

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data is independent and identically distributed (i.i.d) or comes from autocorrelated distribution. The wrong determination will eventually cause the increasing of false observation in process monitoring. This coupled with the existence of error inherent in the model where it gives effect to the accuracy of model estimation.

Inaccurate estimation can be solved using the approach introduced by Efron and Tibshirani (1993), which is hybrid single bootstrap method onto model of study. For first introduction, bootstrap known as sampling the random data,  $X$  with replacement using a number of bootstrap replication i.e.  $B$ , for instance,  $B = 1000$  to estimate the interested inference function distribution of sample data  $X$ . This sampling with replacement method helps to solve the low order of accuracy problem in confidence intervals and bias reduction (Scholz, 1994; Davidson & MacKinnon, 2006). Thus, the introduction of the bootstrap and its application helps to improve the Jackknife method.

The impact of bootstrap study clearly is seen in its application to small data and still relevant to be used until today. However, studies on the double bootstrap has been developed, which is a few years after the single bootstrap introduced. Several studies have been conducted using the double bootstrap, see for example, Scholz (1994), Nankervis (2005), and Davidson and MacKinnon (2006), where the objective of those studies is to increase the accuracy of estimation of bias, one-sided confidence interval and two-sided confidence interval. The double bootstrap helps reduce the error by a factor of order from  $n^{-1}$ , i.e. single bootstrap to order  $n^{-2}$ .

However, most of previous study seems to have computationally expensive and required a long term period to perfect the algorithm of second bootstrap replication,  $BB$ , where typically the resample is made at each of the first bootstrap replication samples,  $B_i$ . For example, the number of first replication,  $B = 1000$ , and the algorithm of double bootstrap replication, let say the  $BB = 2000$ , has to be done in every of  $B = B_1, B_2, \dots, B_{1000}$ , thus from this process, it will eventually have several bundle of double bootstrap replication. It was quite impressive to have a great replication algorithm, but with this complex algorithm, a need of high speed computational is required to reduce the consumed timing and to avoid the computational error. To overcome this typical problem, in this study, an alternative algorithm of double bootstrap constructed. The basic idea is to repeat the same algorithm from single bootstrap, which was hybrid on interested model, i.e. in this study a SPC base model will be used. For example, the  $B = 1000$ , and the algorithm of double bootstrap can be done using the residual from single bootstrap model to perform the second replication, let say  $BB = 2000$ . This new algorithm seems to be easier to be processed and can speed the further computational and model estimation. Furthermore, it will eventually reduces the computational error due

to the use of single algorithm process compare to typical algorithm where it was used the multi repetition process.

Moreover, in this study, an alternative SPC base model of chart is used, i.e. Moving Centerline Exponentially Weighted Moving Average (MCEWMA). It as first introduced by Montgomery (2005) where the model is actually the basic model of Exponentially Weighted Moving Average (EWMA) chart and it can be differs from its ability of limit control. The limitation in this study is it only involves the model of chart and its performance of error and interval estimation. The estimation of limit control is not included and the performance of chart monitoring is not estimated.

### **Objective of the Study**

The main objective of this study is to construct a standard error of confidence interval of MCEWMA mode using the double bootstrap approach. The alternative algorithm is used and eventually this hybrid process coins a new model, i.e. BB-MCEWMA. The practical application of the double bootstrap model, BB-MCEWMA is used to estimate the volatility of return from sukuk investment by Rantau Abang Capital Berhad, Malaysia.

Considering the two-sided confidence interval, the standard interval estimation such as normal and student-t is used. Meanwhile, the bootstrap interval, i.e. biased-corrected and accelerate, BCa from Efron and Tibshirani (1993) also considered in this study. Moreover, the performance of the BB-MCEWMA in reducing the estimation of error, i.e. mean squared error, MSE and root of mean squared error, RMSE is observed in this study, because the accuracy of the new model can be describe through the error estimation.

## **2. LITERATURE REVIEW**

Investment sample data, for example return of sukuk characterised as non-i.i.d sample data and often known as autocorrelated data where current observation data is correlate to previous observation data,  $i-1$  (Ramjee, Crato & Ray, 2002). This kind of data used in SPC and risk management for estimating the investment volatility, and shares the same model estimation, which is EWMA base model. More specifically, in SPC, the EWMA model is given special attention in monitoring process of volatility where this model coupled with expected one-step-ahead limit control. This combination results into charts MCEWMA (Montgomery, 2005; Nembhard & Nembhard, 2000; O'shaughnessy & Haugh, 2002).

Most of previous study canters around the single bootstrap approach on interest model in various field of study, for example, in econometric, where it is

used to reduce inherent errors in the estimated VaR models (Pascual, Romo & Ruiz, 2002; Chiou, hung & Hseu, 2008; Lee, Chiu & Cheng, 2010). The impact of further single bootstrap study is the construction of iterated bootstrap method, where bootstrap sampling is carried out on each sample of first bootstrap replication (Scholz, 1994; Nankervis, 2005). The second iteration bootstrap has lowered order of bias and interval. Despite of successful decrease the error to be factor of order  $n^{-2}$ , the algorithm is complexes and time consumed to be applied.

The performance of double bootstrap model is usually measured by intervals and point estimation. The previous study has found that the percentile bootstrap has got much attention for bootstrap interval method, for example see Jeong and Maddala (1996) and Tong et al (2002), where the study centres on the coverage of interval either in single or double bootstrap. Meanwhile, Nankervis (2005) has used standard interval estimation, which is student-t interval to compare the better performance of coverage interval. Moreover, the error estimation is used to test the performance of model based on theory of smaller error estimation gives more accuracy of model estimation and has a good performance. As for example, O'shaughnessy dan Haugh, (2002) has used the Mean Squared Deviation (MSD) and Root Mean Squared Deviation (RMSD) to estimate the performance of MCEWMA model. While, Wang and Han (2013) has used the Mean Squared Error (MSE) to have the performance of chart model and also state that model that minimizes the MSE estimation be a good performance model.

### 3. RESEARCH MODEL

Consider the MCEWMA model and its residual

$$W_i = \lambda x_i + (1 - \lambda)W_{i-1}, \quad (1)$$

$$e_i = x_i - W_{i-1}, \quad (2)$$

where  $X_i = \{x_1, \dots, x_n\}$  is individual observation of mean process where the initial starting,  $W_0 = \mu_0$ . Let  $\lambda \in (0,1)$  denote the estimation of optimization of  $\lambda$  (Cox, 1961), thus estimation of model (1) and its residual (2) are  $\hat{W}_i = \hat{\lambda}x_i + (1 - \hat{\lambda}) \hat{W}_{i-1}$  and  $\hat{e}_i = x_i - \hat{W}_{i-1}$  respectively.

Double bootstrapping process begins with getting  $B$  single bootstrap replicate, i.e. first bootstrapping process. Consider the estimated residual of single bootstrap model,  $\hat{e}_i^* = \{\hat{e}_i^*, \dots, \hat{e}_n^*\}$ :

$$\hat{Bias}_B = \frac{\sum_{i=1}^n [E(\hat{e}_i^*) - \hat{e}_i^*]}{B} \quad (3)$$

where  $B$  is the number of bootstrap. The replication procedure is repeated until calculation of bias is constant to a certain value, say  $k_1$  and the number of replication,  $B$  is saved. Meanwhile, for double bootstrap replication,  $BB$ , the same process is done using the sample data of estimated residual of double bootstrap model,  $\hat{e}_i^{**} = \{\hat{e}_i^{*(1)}, \dots, \hat{e}_i^{*(B)}\}$ . The replication of  $BB$  is saved when the bias approached and constant to a value,  $k_2$ .

The single bootstrap process can be preceded using the estimation of residual (2) and the sampling with replacement can be done using the first algorithm:

**Step 1:** First resampling process is used using the number replication of single bootstrap,  $B$ , for example  $B = 1500$ . Thus the sample of residual can be defined as  $\hat{e}_i^{*(t)}$ :

$$\hat{e}_i^{*(t)} = \begin{bmatrix} \hat{e}_1^{*(1)}, & \dots, & \hat{e}_1^{*(1500)} \\ \vdots & & \vdots \\ \hat{e}_n^{*(1)}, & \dots, & \hat{e}_n^{*(1500)} \end{bmatrix} \quad (4)$$

**Step 2:** Using the result in Step 2, construct a new data, namely,  $x_i^{*(t)}$  :

$$x_i^{*(t)} = \begin{bmatrix} x_1^{*(1)}, & \dots, & x_1^{*(1500)} \\ \vdots & & \vdots \\ x_n^{*(1)}, & \dots, & x_n^{*(1500)} \end{bmatrix} \quad (5)$$

**Step 3:** Average the row matrices of  $x_i^{*(t)}$  to create a set of single bootstrap sample data,  $X_i^*$ .

The single bootstrap model, BMCEWMA can be estimated using the single bootstrap sample data  $X_i^* = \{x_i^*, \dots, x_n^*\}$ , thus the hybrid model can be given by:

$$W_i^* = \lambda x_i^* + (1 - \lambda)W_{i-1}^* \quad (6)$$

Considering the estimates parameter,  $\hat{\lambda}^B$  using (Cox, 1961) thus, the estimation od BMCEWMA  $\hat{W}_i^* = \hat{\lambda}^B x_i^* + (1 - \hat{\lambda}^B)W_{i-1}^*$  and its residual  $\hat{e}_i^* = x_i^* - \hat{W}_{i-1}^*$ . The second bootstrap approach is to repeat the same steps of first algorithm by using the bootstrap residual of BMCEWMA model. Thus second algorithm can be given by:

**Step 1:** Sampling with replacement process is done using the replication of double single bootstrap,  $BB$ , let say,  $BB = 2000$  to have a set of matrix of residual sample,  $\hat{e}_i^{**}$ :

$$\hat{e}_i^{**}(t) = \begin{bmatrix} \hat{e}_1^{**}(1), & \dots, & \hat{e}_1^{**}(2000) \\ \vdots & & \vdots \\ \hat{e}_n^{**}(1), & \dots, & \hat{e}_n^{**}(2000) \end{bmatrix} \quad (6)$$

**Step 2:** Continue using the  $\hat{e}_i^{**}(t)$  result in Step 2, construct a new data, namely,  $x_i^{**}(t)$  for double bootstrap method:

$$x_i^{**}(t) = \begin{bmatrix} x_1^{**}(1), & \dots, & x_1^{**}(2000) \\ \vdots & & \vdots \\ x_n^{**}(1), & \dots, & x_n^{**}(2000) \end{bmatrix} \quad (7)$$

**Step 3:** Repeated the Step 3 of single bootstrap method, to create a set of double bootstrap sample,  $X_i^{**} \{x_i^{**}, \dots, x_n^{**}\}$ .

Continued with the second algorithm above to estimate the parameter of  $\hat{\lambda}^{BB}$  and the BBMCEWMA model,  $\hat{W}_i^{**} = \hat{\lambda}^B x_i^{**} + (1 - \hat{\lambda}^B) \hat{W}_{i-1}^{**}$  and its residual model,  $\hat{e}_i^{**} = x_i^{**} - \hat{W}_{i-1}^{**}$ .

### Performance Estimation

In this study, two types interval estimation considered to use, i.e. standard interval and bootstrap interval that introduced by Efron and Tibshirani (1993). For standard interval, normal and student-t are suggested to be used, meanwhile the Biased Corrected and accelerate (BCa) is considered for bootstrap interval estimation.

Consider sample data of double bootstrap,  $X_i^{**}$  and the calculation its standard error can be given by:

$$\left[ (\sigma^{**})^2 \cdot \frac{1}{n} \right]^{\frac{1}{2}} \quad (8)$$

and the upper and lower of standard interval can be estimate using the equation given by:

$$[\hat{\theta}_L^{**}, \hat{\theta}_U^{**}] = [\hat{\theta}^{**} - z^\alpha \cdot \hat{E}^{**}, \hat{\theta}^{**} + z^\alpha \cdot \hat{E}^{**}] \quad (9)$$

$$[\hat{\theta}_L^{**}, \hat{\theta}_U^{**}] = [\hat{\theta}^{**} - t_{n-1}^\alpha \cdot \hat{E}^{**}, \hat{\theta}^{**} + t_{n-1}^\alpha \cdot \hat{E}^{**}] \quad (10)$$

where Equation (9) and Equation (10) refers to the normal and student-t interval respectively. While  $\hat{\theta}$  refer to estimation of average of double bootstrap sample,  $X_i^{**}$ , the  $\alpha = 0.05$  used in this study and  $\hat{E}^{**}$  is equally to Equation (8).

Moreover, the estimation for BCa interval, basically, can be given by:

$$\left[ \hat{\theta}_{L(BCa)}^{**}, \hat{\theta}_{U(BCa)}^{**} \right] = \left[ \hat{\theta}^{**(\alpha_1)}, \hat{\theta}^{**(\alpha_2)} \right] \tag{11}$$

where  $\hat{\theta}_{L(BCa)}^{**}$  and  $\hat{\theta}_{U(BCa)}^{**}$  a refers to the upper and lower interval of double bootstrap respectively, and  $\alpha_{j=1,2} = \{\alpha_1, \alpha_2\}$  is estimated by equation given by:

$$\alpha_1 = \Phi \left( \hat{z}_0 + \frac{\hat{z}_0 + z^\alpha}{1 - \hat{a}(\hat{z}_0 + z^\alpha)} \right) \text{ and } \alpha_2 = \Phi \left( \hat{z}_0 + \frac{\hat{z}_0 + z^{(1-\alpha)}}{1 - \hat{a}(\hat{z}_0 + z^{(1-\alpha)})} \right) \tag{12}$$

where  $a$  is accelerate value, while the notion  $z^\alpha$  refer to the normal confidence interval with  $\alpha = 0.05$  and  $\Phi(\cdot)$  and is cumulative distribution function (c.d.f). When the estimation of  $\hat{a}$  and  $\hat{z}_0$  equals to zero value, thus the Equation (12) can be rewrite as  $\alpha_1 = \Phi(z^{0.95}) = \alpha$ , and  $\alpha_2 = \Phi(z^{0.95}) = 1 - \alpha$ .

The accuracy of double bootstrap model also asses with point estimator Mean Square Error (MSE) and Root Mean Square Error (RMSE) will be applied. Considering the residual of double bootstrap model,  $\hat{e}_i^{**}$  then the point estimator can be given by:

$$MSE = \frac{\sum_{i=1}^n \left[ \hat{e}_i^{**} - E(\hat{e}^{**}) \right]^2}{n} \tag{13}$$

where  $\hat{e}^{**}$  comes from residual of double boostsrat model, and  $n$  refers to saiz of sampel used. The last method is, RMSE. Where it is used to estimate square root of the average of difference between the actual sample data and one-step-ahead prediction model. In this study, the result of performance will be focus on model that minimized RMSE method. Those models that give the minimal value can be categorized as a good performer. The RMSE method can be writing as below:

$$RMSE = \sqrt{\frac{\sum_{i=1}^n \left[ \hat{e}_i^{**} - E(\hat{e}^{**}) \right]^2}{n}} \tag{14}$$

this estimation form also can be applicable for single bootstrap approach model, by using value of  $\hat{e}_i^*$  and  $E(\hat{e}^*)$ . For comparison of performance, the original and single bootstrap model also estimated in this study. The result can be refers to the next section.

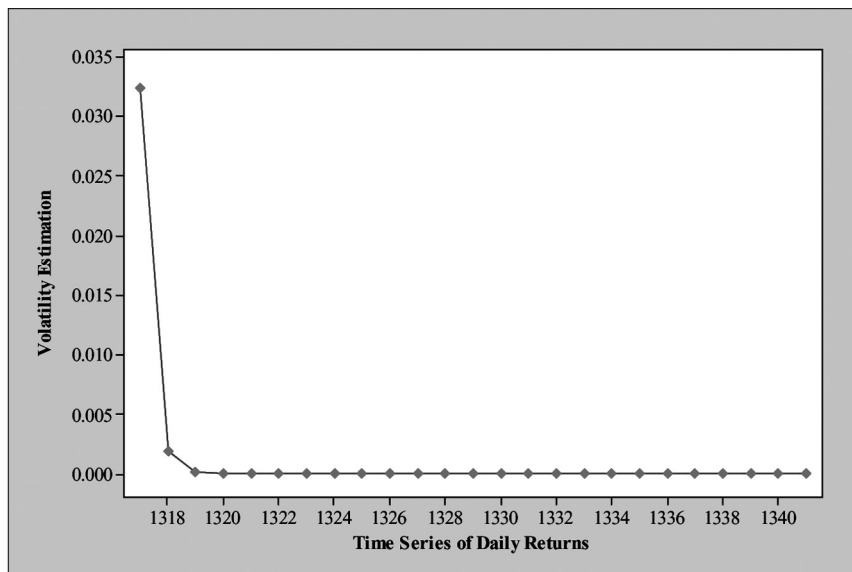
#### 4. DATA ANALYSIS

In this section, this study applies the algorithm of double bootstrap model, BB-MCEWMA to construct the prediction of estimation of the Rantau Abang Capital Berhad sukuk investment, VI060188. The estimation of BB-MCEWMA model used for the prediction of future returns and volatilities is based on daily closing prices of VI060188 observed from 15 March 2006 to 3 March 2011, with a total of 2016 observations. As unavoidably, daily returns,  $X_{i=1, \dots, n}$  are obtained as first differences of logarithms, which is  $\log(S_i/S_{i-1})$  where  $S_i$  denoted the closing price at day  $i = 1, \dots, n$ . Using these returns, the parameter of original model, MCEWMA is estimated and the value finds that  $\hat{\lambda} \approx 0.94$  as the best selection for daily returns. Thus, the prediction of MCEWMA model can be given by:

$$\hat{W}_{i+1} = 0.94 x_{i+1} + (0.06)\hat{W}_i \quad (15)$$

Estimation of volatility model of (15) can be referred on Figure 1, where the volatility versus times series of sukuk Musyarakah's daily returns data, i.e. the data is observed only starting from data 1317 until data 1341. The main reason for observing the range of data is because to observe the fluctuation happened in the middle of sukuk issuance. Based on the Figure 1, it is shown that the sukuk has low volatility, where using the original model, MCEWMA, the higher estimation is on data of 1317, and continued to be lower than volatility of 0.005.

**Figure 1: Estimation of original volatility model, MCEWMA using the daily returns of sukuk Musyarakah**





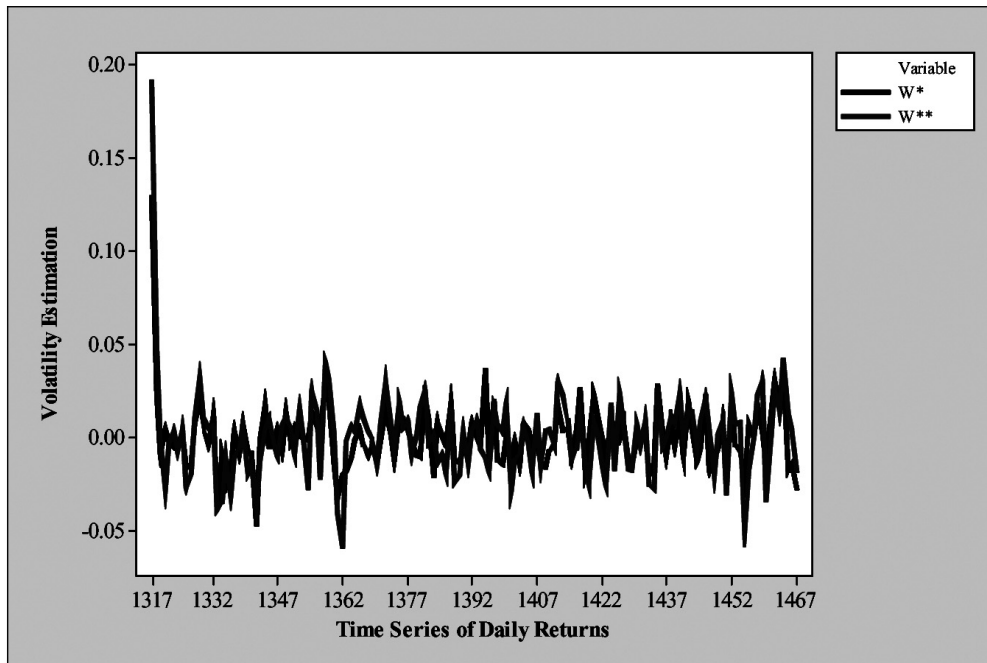
By applying the second algorithm, the prediction of double bootstrap model, BB-MCEWMA can be given by:

$$\hat{W}_{i+1}^{**} = 0.94 x_{i+1}^{**} + (0.06)\hat{W}_i^{**} \tag{16}$$

and can be graphically shown in Figure 2, where the single bootstrap model of volatility also plotted on the same figure. The time series of daily returns is observed on data 1317 until data1467. The reason is to see the occurred fluctuation of sukuk volatility when using the estimation of bootstrap model.

Based on Figure 2, it is shown that using the bootstrap model a high volatility happened, where the fluctuation occurred dramatically, for example double bootstrap model, BB-MCEWMA, the volatility estimation of data 1347 is 0.00793, and increased to 0.0011206 (data 1348) and dramatically decreased to 0.004096 (data 1349).

**Figure 2: Estimation of bootstrap volatility model, BMCEWMA and BB-MCEWMA using the daily returns of sukuk Musyarakah**



Referring the algorithm for model (14), the first bootstrapping process draws a sample from residual of model (15) using  $B = 2150$  replication and continue to the second algorithm in previous section, the second replication,  $BB$  equal to 2125. The

performance of estimation of volatility models, which are original (MCEWMA), single bootstrap (BMCEWMA) and double bootstrap (BB-MCEWMA) are observed by interval estimation and point estimation. The main concern of this performance is to find the better model estimation of volatility, where it can be shown by the shortest length of interval and smallest error of point estimation. Theoretically, the shortest length shows that the model estimation is more approach to the true value, while the smallest error state that the model is more accurate.

**Table 1**  
**Length of Standard and Bootstrap Interval Estimation**

<i>Model</i>	<i>Estimation Value</i>					
	<i>Normal</i>	<i>(%)<sup>a</sup></i>	<i>Student-t</i>	<i>(%)</i>	<i>BCa</i>	<i>(%)</i>
MCEWMA	0.052835		0.052859		b	
BMCEWMA	0.051209	0.163 <sup>c</sup>	0.051233	0.163	0.00885	
BB-MCEWMA	(0.049744)	0.309 <sup>d</sup>	(0.049767)	0.309	(0.00873)	0.013

<sup>a</sup>Percentage of decreasing length of models. <sup>b</sup>The BCa interval method is specifically used to estimate the bootstrap sample. <sup>c</sup>The difference length of MCEWMA and single bootstrap model. <sup>d</sup>The difference between MCEWMA and BB-MCEWMA.

Table 1 presents the length of standard and bootstrap interval estimation for model MCEWMA, BMCEWMA and BB-MCEWMA. The result for the MCEWMA model shows longest length estimation either in normal and student-t interval. Meanwhile the single bootstrap approach decrease the length value estimation of standard interval method, for example normal interval, is 0.051209 where the decreasing percent is about 0.163% (differences of MCEWMA and BMCEWMA). However, the double bootstrap model gives a better performance on interval estimation where BB-MCEWMA model gives the length of 0.049744, i.e. normal interval, and percentage of decreasing is 0.309% (MCEWMA to BB-MCEWMA) and 0.147% (BMCEWMA to BB-MCEWMA).

The length of standard and bootstrap interval estimation, as graphically displayed in Figure 3, decrease with bootstrapping approach to the original model, MCEWMA. The student-t in Figure 3(i), for example, smoothly decreases from {0.052859 → 0.051233} and dramatically decreases to 0.049767 at BB-MCEWMA model. Meanwhile, the BCa method in Figure 3(ii) shows small decreasing length, i.e. from BMCEWMA to BB-MCEWMA. The percentage of decreasing is only 0.013%, refer to Table 1.

Figure 3: Interval Estimation of (i) Standard method and (ii) Bootstrap method

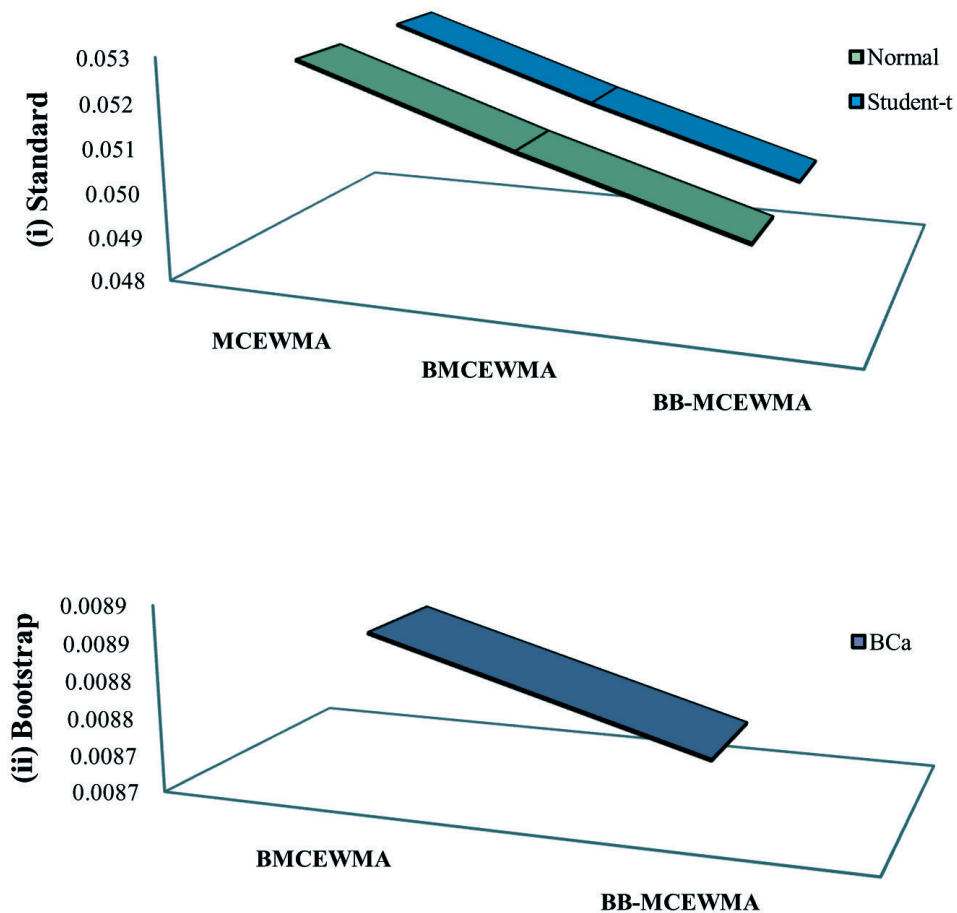


Table 2, shows the point estimator performance of BB-MCEWMA including the original model and single bootstrap. The value of error estimation decrease slowly with the hybrid process, for example MSE, the decreasing estimation starts with  $\{0.523237 \rightarrow 0.463612 \rightarrow 0.413579\}$ . The decreasing percentage of MSE shows a positive result for double bootstrap model, where the differences of original model to BB-MCEWMA is 10.965% while for single bootstrap model to BB-MCEWMA is 5.003%. The small percentage of BMCEWMA to BB-MCEWMA apparently due to the small repetition *BB*, i.e. 2125 compare to single bootstrap, *B*.

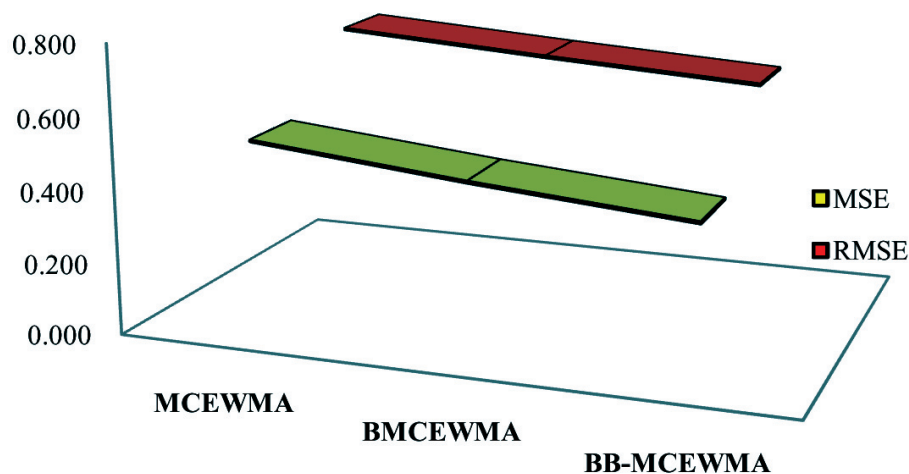
**Table 2**  
**Error Estimation**

Model	Estimation Value of Efficiency			
	MSE	(%) <sup>a</sup>	RMSE	(%)
MCEWMA	0.523237		0.723351	
BMCEWMA	0.463612	5.962 <sup>b</sup>	0.680891	4.246
BB-MCEWMA	(0.413579)	10.965 <sup>c</sup> (5.003) <sup>d</sup>	(0.643101)	8.025 (3.779)

<sup>a</sup>Percentage of decreasing error estimation. <sup>b</sup>The difference error value of MCEWMA and single bootstrap model. <sup>c</sup>The difference between MCEWMA and BB-MCEWMA. <sup>d</sup>The percentage of decreasing error value of BMCEWMA and BB-MCEWMA.

Similar pattern result of decreasing percentage shows by RMSE estimation, where the differences of original model to BMCEWMA and BB-MCEWMA are 4.246% and 8.025% respectively. The percentage also highlights the small percentage of decreasing RMSE estimation from single bootstrap model to double bootstrap model, i.e. 3.779% compare to the higher decreasing of first bootstrap approach process, which is 4.246%. It is apparently to the same reason of MSE estimation, i.e.  $BB < B$ . The estimation value of MSE and RMSE are graphically displayed in Figure 4.

**Figure 4: Error Estimation Using MSE and RMSE**



Referring the Figure 4, the RMSE estimation value of efficiency shows the decreasing estimation starting from MCEWMA to BMCEWMA to BB-MCEWMA. The double bootstrap approach has increased the accuracy of original model estimation, where the RMSE estimation decreases from {0.723351 → 0.643101}. Meanwhile, the single bootstrap shows small decreasing from MCEWMA (0.723351) to BMCEWMA (0.680891). Thus, by considering both point estimation method, it is proven that double bootstrap approach fixed and given high accuracy of model estimation.

## 5. DISCUSSION

The application of sukuk data demonstrates the performance that can happen when the standard error of MCEWMA confidence interval hybrid with double bootstrap method. As hybrid process increase, the length of interval estimation decreased, and often yielding the dramatically shortest length of BMCEWMA and BB-MCEWMA of standard interval. The decreasing length of BCa method from single bootstrap model to double bootstrap model given a small effect because it's directly relate to the number of replication double bootstrap. In MSE and RMSE estimation, the percentage of efficiency found to be up to 4% to 10% efficiency of MCEWMA differs to single bootstrap model and double bootstrap model. At BB-MCEWMA model, the percentage of MSE and RMSE that differs from original model are 10.965% and 8.025% respectively. However, it's found that, because of small value of replication *BB*, the difference percentage of BMCEWMA and BB-MCEWMA only 5.003% (MSE) and 3.779% (RMSE).

This study has statistically proven, i.e. shown by the estimation model performance, that the double bootstrap method improved the original model estimation. As state by Nankervis (2005), the double bootstrap helps to reduce the biasness and length of interval in the model estimation. Even though, use the alternative double bootstrap algorithm, the length of interval is eventually decreased and the percentage of decreasing is a great deal comparing to original model. Meanwhile, Davidson and MacKinnon (2006) state that the error in the model is reduced using the double bootstrap method and this statement is proven in this study, where the result of MSE and RMSE is found to be smallest compare the other two models.

## 6 CONCLUSION

In this study, a MCEWMA model was hybrid with alternative algorithm of double bootstrap method. As for the typical double bootstrap, it was found to be complexes and time consumed to be applied. The hybrid process was to construct a standard error of MCEWMA confidence interval and eventually coins a new

hybrid model, BB-MCEWMA. The BB-MCEWMA model basically was one of the SPC control chart. The limitation of this study was to test the base model only and the application was to estimate the prediction of sukuk volatility and the performance of model was tested using two kinds of estimation, i.e. interval and error estimation. Using returns from sukuk and the volatility of sukuk data, the BB-MCEWMA gave a better performance in all interval estimation, which was showed shortest length of normal, student-t and BCa method. Moreover, in point estimation, the BB-MCEWMA also showed the better performance of MSE and RMSE, where the error estimation was smallest compared to original model and single bootstrap model. However, an interesting result found that, a low percentage performance of BB-MCEWMA model differs to BMCEWMA. It was understand that the performance of BB-MCEWMA model depended to the number of double bootstrap replication. For further study, it was recommended to used a greater number of double bootstrap replication, i.e.  $BB > B$  for observing the model estimation performance. Despite of low percentage performance, the BB-MCEWMA model gave accurate expected estimation of sukuk volatility and more efficient to be used. Thus, to complete this whole study, in terms of quality control, the upper and lower limit control chart is recommended to be estimated. The control chart can be known as BB-MCEWMA chart, and the monitoring of sukuk volatility can be well described either it was in-control or happened to be out-of-control.

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