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FUZZY SYSTEMS AND DATA MINING II

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A New Integrating SAW-TOPSIS Based on Interval Type-2 Fuzzy Sets for Decision Making

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Abstract. Most of the integrated methods of multi-attributes decision making (MADM) used type-1 fuzzy sets to represent uncertainties. Recent theory has suggested that interval type-2 fuzzy sets (IT2 FS) could be used to enhance representation of uncertainties in decision making problems. Differently from the typical integrated MADM methods which directly used type-1 fuzzy sets, this paper proposes an integrating simple additive weighting - technique for order preference similar to ideal solution (SAW-TOPSIS) based on IT2 FS to enhance judgment. The SAW with IT2 FS is used to determine the weight for each criterion, while TOPSIS method with IT2 FS is used to obtain the final ranking for the attributes. A numerical example is used to illustrate the proposed method. The numerical results show that the proposed integrating method is feasible in solving MADM problems under complicated fuzzy environments. In essence, the integrating SAW-TOPSIS is equipped with IT2 FS in contrast to type-1 fuzzy sets for solving MADM problems. The proposed method would make a great impact and significance for the practical implementation. Finally, this paper provides some recommendations for future research directions.

Keywords. Interval type-2 fuzzy set, Simple additive weighting, Multi-criteria decision making , TOPSIS, preference order

Introduction

Decision making based on multi-criteria evaluation has been used with great success for many applications. Most of these applications are characterized by high levels of uncertainties and vague information. Fuzzy set theory has provided a useful way to deal with vagueness and uncertainties in solving multi-criteria decision making (MCDM) problem. During the last two decades, MCDM methods that integrated with fuzzy sets have been one of the fastest growing research areas. Abdullah [1] presents a brief review of category in the integration of fuzzy sets and MCDM. In general, MCDM can be categorized into multi-attribute decision making (MADM) and multi-objective decision making (MODM). Naturally, MADM problem is related to multiple attributes. The attributes of MADM represent the different dimensions from which the alternatives can be viewed by decision makers. There are many fuzzy MADM methods that have been discussed in the literature, and fuzzy technique for order preference

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similar to ideal solution (FTOPSIS) is one of the MADM methods. Preference or decision derived from FTOPSIS is made by observing the degree of closeness to ideal solution. Add to this method, fuzzy simple additive weighting (FSAW) is another type of fuzzy MADM methods. It is an extension of the SAW method, where it employs trapezoidal fuzzy numbers to represent imprecision in judgements.

Lately, the integration of MADM method has received considerable attention in literature. Integrated method is simply defined as two or more methods that are concurrently employed to solve decision making problems. For example, the TOPSIS is integrated with fuzzy analytic hierarchy process (FAHP) model to propose a new integrated model for selecting plastic recycling method [2]. Rezaie et al., [3] present an integrating model based on FAHP and VIKOR method for evaluating cement firms. Wang et al., [4] develop an integrating OWA–fuzzy TOPSIS to tackle fuzzy MADM problems. Kharat et al., [5] applied an integrated fuzzy AHP–TOPSIS to municipal solid waste landfill site selection problem. Pamučar and Ćirović [6] applied the new integrated fuzzy DEMATEL–MABAC method in making investment decisions. Tavana et al., [7] proposed an integrated fuzzy ANP-COPRAS-Grey method to determine the selection of social media platform.

Most of these integrating methods employed type-1 fuzzy sets to represent uncertainties in decision making. However, the type-1 fuzzy sets have some extent of limitation in dealing with uncertainties. Recent theories suggest that interval type-2 fuzzy sets (IT2 FSs) are more flexible than the interval type-1 fuzzy sets in representing uncertainties. Therefore, in contrast to these methods, this paper introduces linguistic terms based on IT2 FS for proposing a new integrating MADM method. The IT2 FS is incorporated within the framework of FSAW and FTOPSIS to develop a new integrating fuzzy MADM method. Specifically, Interval Type-2 Fuzzy Simple Additive Weighting (IT2 FSAW) method is integrated with Interval Type-2 Technique for Order Preference Similar to Ideal Solution (IT2 FTOPSIS) method for solving MADM problems. In the proposed method, the judgements made by decision makers over the relative importance of alternatives are determined using IT2 FSAW procedure and the final preference is obtained using IT2 FTOPSIS. The ranking method of IT2 FTOPSIS approach preserves the characteristics of fuzzy numbers where the linguistic terms can easily be converted to fuzzy numbers.

1. Proposed Method

This paper integrates the IT2 FSAW with IT2 FTOPSIS to establish a new MADM method. In this proposed method, the IT2 FSAW is used to find weights of the criteria, whereas IT2 FTOPSIS is used to establish preference of alternatives. The definitions of IT2 FS [8], upper and lower memberships of IT2 FS [9], and ranking values of the trapezoidal IT2 FS [10] are used in the proposed method. The detailed procedure of the proposed method is described as follows.

Step 1: Construct the decision matrix Y_p of the p-th decision maker and construct the average decision matrix $\overline{Y}$, respectively.

$$Y_p = (\tilde{\tilde{f}}_{ij}^{p})_{m \times n} = \begin{array}{c} \\ f_1 \\ f_2 \\ \vdots \\ f_m \end{array} \overset{\begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array}}{\begin{bmatrix} \tilde{\tilde{f}}_{11}^{p} & \tilde{\tilde{f}}_{12}^{p} & \cdots & \tilde{\tilde{f}}_{1n}^{p} \\ \tilde{\tilde{f}}_{21}^{p} & \tilde{\tilde{f}}_{22}^{p} & \cdots & \tilde{\tilde{f}}_{2n}^{p} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{f}}_{m1}^{p} & \tilde{\tilde{f}}_{m2}^{p} & \cdots & \tilde{\tilde{f}}_{mn}^{p} \end{bmatrix}}$$

$$\bar{Y} = (\tilde{\tilde{f}}_{ij})_{m \times n}, \tag{1}$$

where $\tilde{\tilde{f}}_{ij} = \left(\dfrac{\tilde{\tilde{f}}_{ij}^{1} \oplus \tilde{\tilde{f}}_{ij}^{2} \oplus \ldots \oplus \tilde{\tilde{f}}_{ij}^{k}}{k} \right)$, $f_1, f_2, \ldots, f_m$ represent the criteria and $x_1, x_2, \ldots, x_n$ represents alternatives.

Step 2: Construct the aggregated fuzzy weight $\bar{W}$, from the weighting matrix W_p of the attributes provided by p-th decision maker.

Let $\tilde{\tilde{w}}_i^{p} = (a_i, b_i, c_i, d_i), \quad i = 1, 2, \ldots, m$ be the linguistic weight given to the subjective criteria $C_1, C_2, \ldots, C_h$ and objective criteria $C_{h+1}, C_{h+1}, \ldots, C_n$ by decision maker D_t.

$$W_p = (\tilde{\tilde{w}}_i^{p})_{1 \times m} = \overset{\begin{array}{cccc} f_1 & f_2 & \cdots & f_m \end{array}}{\begin{bmatrix} \tilde{\tilde{w}}_1^{p} & \tilde{\tilde{w}}_2^{p} & \cdots & \tilde{\tilde{w}}_m^{p} \end{bmatrix}}, \tag{2}$$

$$\bar{W} = (\tilde{\tilde{w}}_i)_{1 \times m}, \tag{3}$$

where $\tilde{\tilde{w}}_i = \dfrac{\tilde{\tilde{w}}_i^{1} \oplus \tilde{\tilde{w}}_i^{2} \oplus \tilde{\tilde{w}}_i^{k}}{k}$, $\tilde{\tilde{w}}_i$ is an interval type-2 fuzzy set.

To defuzzify weights of fuzzy attribute, the signed distance is employed [11].
Defuzzification of $\bar{W}$ is represented as:

$$d(\bar{W}_j) = \frac{1}{4}(\tilde{\tilde{w}}_j^{1} + \tilde{\tilde{w}}_j^{2} + \tilde{\tilde{w}}_j^{3} + \tilde{\tilde{w}}_j^{4}), \quad j = 1, 2, \ldots, n \tag{4}$$

The crisp value for criteria $\tilde{\tilde{W}}$. is given by:

$$\tilde{\tilde{W}}_j = \frac{d(\bar{W}_j)}{\sum_{j=1}^{n} d(\bar{W}_j)}, \; j = 1, 2, \ldots, n \tag{5}$$

where $\sum_{j=1}^{n} \tilde{\tilde{W}}_j = 1$. Therefore, the weight vector $W = [\tilde{\tilde{W}}_1, \tilde{\tilde{W}}_2, \ldots, \tilde{\tilde{W}}_n]$ is constructed.

Step 3: Create the weighted decision matrix $\bar{Y}_w$,

$$\tilde{\tilde{Y}}_w = \left(\tilde{\tilde{v}}_{ij}\right)_{m\times n} = \begin{array}{c} \\ f_1 \\ f_2 \\ \vdots \\ f_m \end{array}\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} \tilde{\tilde{v}}_{11} & \tilde{\tilde{v}}_{12} & \cdots & \tilde{\tilde{v}}_{1n} \\ \tilde{\tilde{v}}_{21} & \tilde{\tilde{v}}_{22} & \cdots & \tilde{\tilde{v}}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{v}}_{m1} & \tilde{\tilde{v}}_{m2} & \cdots & \tilde{\tilde{v}}_{mn} \end{bmatrix} \end{array}, \tag{6}$$

where $\tilde{\tilde{v}}_{ij} = \tilde{\tilde{W}} \otimes \tilde{\tilde{f}}_{ij}$, $1 \le i \le m$, and $1 \le j \le n$.

Step 4: Calculate the ranking value $Rank\left(\tilde{\tilde{v}}_{ij}\right)$ of the IT2 FS $\tilde{\tilde{v}}_{ij}$, using Eq (7). Create the ranking for weighted decision matrix $\tilde{\tilde{Y}}_w^*$.

$$\tilde{\tilde{Y}}_w^* = \left(Rank\left(\tilde{\tilde{v}}_{ij}\right)\right)_{m\times n}, \tag{7}$$

Step 5: Calculate the positive-ideal solution $x^+ = \left(v_1^+, v_2^+, \ldots, v_m^+\right)$ and the negative-ideal solution $x^- = \left(v_1^-, v_2^-, \ldots, v_m^-\right)$, where

$$v_i^+ = \begin{cases} \max\limits_{1\le j\le n}\{Rank\left(\tilde{\tilde{v}}_{ij}\right)\}, & \text{if } f_i \in F_1 \\ \min\limits_{1\le j\le n}\{Rank\left(\tilde{\tilde{v}}_{ij}\right)\}, & \text{if } f_i \in F_2 \end{cases} \tag{8}$$

and

$$v_i^- = \begin{cases} \min\limits_{1\le j\le n}\{Rank\left(\tilde{\tilde{v}}_{ij}\right)\}, & \text{if } f_i \in F_1 \\ \max\limits_{1\le j\le n}\{Rank\left(\tilde{\tilde{v}}_{ij}\right)\}, & \text{if } f_i \in F_2 \end{cases} \tag{9}$$

where F_1 denoted the set of benefit attributes, and F_2 denotes the set of cost attributes.

Step 6: Find the distance $d^+\left(x_j\right)$ between each alternative x_j and the positive ideal solution x^+, using the Eq (10).

$$d^+\left(x_j\right) = \sqrt{\sum_{i=1}^{m}\left(Rank\left(\tilde{\tilde{v}}_{ij}\right) - v_i^+\right)^2}, \tag{10}$$

where $1 \le j \le n$. Similarly, find the distance $d^-\left(x_j\right)$ between each alternative x_j and the negative-ideal solution x^-, using the following equation.

$$d^-\left(x_j\right) = \sqrt{\sum_{i=1}^{m}\left(Rank\left(\tilde{\tilde{v}}_{ij}\right) - v_i^-\right)^2}, \tag{11}$$

Step 7: Calculate the degree of closeness $C\left(x_j\right)$ of x_j with respect to the positive ideal solution x^+, using the following equation.

$$C\left(x_j\right)=\frac{d^{-}\left(x_j\right)}{d^{+}\left(x_j\right)+d^{-}\left(x_j\right)}, \tag{12}$$

where $1 \le j \le n$.

Step 8: Arrange the values of $C\left(x_j\right)$ in a descending order, and the larger value of $C\left(x_j\right)$, indicates the higher preference of the alternative x_j.

2. Numerical Example

For the purpose of illustration and to show the feasibility of the proposed method, an example is presented. This example is retrieved from Chou et al. [5].

Researchers intend to identify the facility location alternatives to build a new plant. The team has identified three alternatives which are alternative 1 (A_1), alternative 2 (A_2), and alternative 3 (A_3). To determine the best alternative site, a committee of four decision makers is created; decision maker 1 (D_1), decision maker 2 (D_2), decision maker 3 (D_3) and decision maker 4 (D_4). Three selection criteria are deliberated: transportation availability (C_1), availability of skilled workers (C_2) and climatic conditions (C_3). Table 1 shows the linguistic terms used to rate criteria with respect to alternatives and also the weights for criteria.

Table 1. Linguistic terms and IT2 FS

Linguistic Terms	Interval Type-2 Fuzzy Sets
Very Poor (VP)	((0,0,0,0.1;1,1),(0,0,0,0.05;0.9,0.9))
Poor (P)	((0.0,0.1,0.1,0.3;1,1),(0.05,0.1,0.1,0.2;0.9,0.9))
Medium Poor (MP)	((0.1,0.3,0.3,0.5;1,1),(0.2,0.3,0.3,0.4;0.9,0.9))
Fair (F)	((0.3,0.5,0.5,0.7;1,1),(0.4,0.5,0.5,0.6;0.9,0.9))
Medium Good (MG)	((0.5,0.7,0.7,0.9;1,1),(0.6,0.7,0.7,0.8;0.9,0.9))
Good (G)	((0.7,0.9,0.9,1;1,1),(0.8,0.9,0.9,0.95;0.9,0.9))
Very Good (VG)	((0.9,1,1,1;1,1),(0.95,1,1,1;0.9,0.9))

Based on the ratings given by decision makers , the example is solved using the proposed method. The final degree of closeness and preference are shown in Table 2.

Table 2. Degree of closeness and preference

Degree of closeness		Preference order
$C(A_1)$	0.4112	3
$C(A_2)$	0.4605	2
$C(A_3)$	0.4778	1

It can be seen that the preference order of the alternatives is A3 $\succ$ A2 $\succ$ A1. The proposed method therefore decided that the best alternative is A3. This preference is slightly inconsistent with the result obtained using the FSAW where the preference is A2 $\succ$ A3 $\succ$ A1.

3. Conclusions

This paper proposed a novel method, which integrate IT2 FSAW and IT2 FTOPSIS to solve MADM problems. Decision makers used interval type-2 linguistic variables to assess the importance of the criterion. The ranking weighted decision matrix obtained from IT2 FSAW was then used as an input to the IT2 FTOPSIS where ideal solutions could be computed. Finally, preference of alternatives was obtained as a result of the implementation using the integration method. To illustrate the feasibility of the proposed method, a numerical example, that formerly solved using the FSAW method was considered. The results showed that A3 is the most preferred alternative. Detailed comparative analysis between the results obtained using the integrated method and other decision making methods is left for future research. Future research may also include sensitivity analysis where the uncertainty of the final preference of the integrating model can be investigated.

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