

NUMERICAL INVESTIGATION ON
IMPULSIVE FUZZY DIFFERENTIAL
EQUATIONS

NOR SHAMSIDAH AMIR HAMZAH

DOCTOR OF PHILOSOPHY
UNIVERSITI MALAYSIA TERENGGANU
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**Thesis Submitted in Fulfilment of the Requirement
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NUMERICAL INVESTIGATION ON NONLINEAR FUZZY DIFFERENTIAL EQUATIONS

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Dedicated To

My beloved family

My parents, Haji Amir Hamzah and Hajjah Rasmah, my mother-in-law, Hajjah Kasum, my husband, Shahibol and my children Shasha, Shahmi, Haziq, Sharah and Hazim.

Thank you

"So far as laws of mathematics refer to reality, they are not certain. And so far as they are certain, they do not refer to reality"

Albert Einstein (1921)

Abstract of thesis presented to the Senate of Universiti Malaysia Terengganu in fulfilment of the requirement for the degree of Doctor of Philosophy

NUMERICAL INVESTIGATION ON IMPULSIVE FUZZY DIFFERENTIAL EQUATIONS

NOR SHAMSIDAH AMIR HAMZAH

August 2012

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Faculty : Science and Technology

Many evolution processes are characterized by the fact that at certain moments of time, they experience a change of state abruptly. It is assume naturally, that those perturbations act instantaneously, in the form of impulses. Thus impulsive differential equations, by means, differential equations involving impulse effects, are seen as a natural description of observed evolution phenomenon of several real world problems. For example, systems with impulse effect have applications in physics, biotechnology, industrial robotics, pharmacokinetics, population dynamics, ecology, optimal control, production theory and many others. Therefore, it is beneficial to study the theory of impulsive differential equations as a well deserved discipline, due to the increase applications of impulsive differential equations in various fields in the future.

However, in many mathematical modeling of the real world problems, fuzziness and impulsiveness occurs simultaneously. This problem could be better modeled by impulsive fuzzy differential equations. Therefore, this research applies the theory of impulsive fuzzy differential equations by combining the theories of impulsive differential equations and fuzzy differential equations. The numerical algorithms are developed and the solutions are verified by comparing the results with the analytical solutions.

The method for the first order linear impulsive fuzzy differential equations under generalized differentiability is also proposed analytically and numerically. Finally, the convergence theorem for the impulsive fuzzy differential equations under generalized differentiability is defined.

Abstrak tesis yang dikemukakan kepada Senat Universiti Malaysia Terengganu
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**SIASATAN BERANGKA KE ATAS PERSAMAAN PEMBEZAAN IMPULSIF
KABUR**

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Ogos 2012

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Kebanyakan proses-proses evolusi dicirikan berdasarkan fakta di mana pada momen-momen waktu tertentu, proses-proses tersebut mengalami perubahan keadaan secara tiba-tiba. Secara semulajadi, anggapan diandaikan bahawa gangguan tersebut bertindak segera dalam bentuk impuls. Sehubungan dengan itu, persamaan pembezaan impulsif atau persamaan pembezaan yang melibatkan kesan impuls, dilihat sebagai penerangan semula jadi yang diperhatikan berlaku ke atas beberapa fenomena evolusi dalam masalah kehidupan dunia sebenar. Sebagai contoh, sistem dengan kesan impuls mempunyai aplikasi dalam fizik, bioteknologi, robot industri, farmakokinetik, populasi dinamik, ekologi, kawalan optima, teori produksi dan banyak lagi. Oleh itu, kajian tentang teori persamaan pembezaan impulsif sebagai satu disiplin amatlah bermanfaat untuk dijalankan memandangkan kepelbagaian aplikasi persamaan pembezaan impulsif dalam pelbagai bidang di masa akan datang.

Walaupun bagaimanapun, dalam pemodelan matematik untuk masalah dunia sebenar, kekaburan dan impuls berlaku serentak. Masalah seperti ini sesuai dimodelkan melalui persamaan pembezaan impulsif kabur. Oleh itu, penyelidikan ini mengaplikasi teori persamaan pembezaan impulsif kabur dengan menggabungkan teori persamaan pembezaan impulsif dan teori persamaan pembezaan kabur. Algoritma berangka dibangunkan dan penyelesaian telah disahkan melalui perbandingan dengan penyelesaian analitik.

Kaedah untuk menyiasat penyelesaian persamaan pembezaan impulsif kabur menggunakan pembezaan general (“generalized differentiability”) telah dicadangkan dan dilaksanakan secara analitik dan berangka. Akhirnya, definisi teorem penumpuan untuk persamaan pembezaan impulsif kabur menggunakan pembezaan general (“generalized differentiability) telah diberikan.