

A Mixture - based Framework for
Nonparametric Density Estimation

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A thesis submitted in partial fulfilment of the requirements
for the degree of Doctor of Philosophy in Statistics,
The University of Auckland,
2011.

Abstract

The primary goal of this thesis is to provide a mixture-based framework for nonparametric density estimation. This framework advocates the use of a mixture model with a nonparametric mixing distribution to approximate the distribution of the data. The implementation of a mixture-based nonparametric density estimator generally requires the specification of parameters in a mixture model and the choice of the bandwidth parameter. Consequently, a nonparametric methodology consisting of both the estimation and selection steps is described. For the estimation of parameters in mixture models, we employ the minimum disparity estimation framework within which there exist several estimation approaches differing in the way smoothing is incorporated in the disparity objective function. For the selection of the bandwidth parameter, we study some popular methods such as cross-validation and information criteria-based model selection methods. Also, new algorithms are developed for the computation of the mixture-based nonparametric density estimates.

A series of studies on mixture-based nonparametric density estimators is presented, ranging from the problems of nonparametric density estimation in general to estimation under constraints. The problem of estimating symmetric densities is firstly investigated, followed by an extension in which the interest lies in estimating finite mixtures of symmetric densities. The third study utilizes the idea of double smoothing in defining the least squares criterion for mixture-based nonparametric density estimation. For these problems, numerical studies whether using both simulated and real data examples suggest that the performance of the mixture-based nonparametric density estimators is generally better than or at least competitive with that of the kernel-based nonparametric density estimators. The last but not least concern is nonparametric estimation of continuous and discrete distributions under shape constraints. Particularly, a new model called the discrete k -monotone is proposed for estimating the number of unknown species. In fact, the discrete k -monotone distribution is a mixture of specific discrete beta distributions. Empirical results indicate that the new model outperforms the commonly used nonparametric Poisson mixture model in the context of species richness estimation. Although there remain issues to be resolved, the promising results from our series of studies make the mixture-based framework a valuable tool for nonparametric density estimation.